

# Quadrature detection, reduced dimensionality and GFT-NMR

GFT NMR, a New Approach To Rapidly Obtain Precise  
High-Dimensional NMR Spectral Information

Seho Kim\* and Thomas Szyperski\*

Contribution from the Department of Chemistry, University at Buffalo, The State University of  
New York, The Northeast Structural Chemistry Consortium, Buffalo, New York 14260

Received August 19, 2002; E-mail: szyperski@chem.buffalo.edu

J. Am. Chem. Soc. 125, 1385-1393 (2003)



## Measuring time in NMR

1D: 32 scans: 42 seconds

2D: 512 FIDs, 16 scans: 10650 sec, 3 hours

3D: 128 x 128 FIDs, 8 scans: 170394 sec, 48 hours

4D: 32 x 32 x 64 FIDs, 4 scans: 340787 sec, 96 hours

oder 512 x 128 x 128 FIDs, 1 scan: 10905190 sec,  
3030 hours, 126 days

We are at the **sampling limit** rather  
than at the **sensitivity limit**



Peter Schmieder

27.05.2004

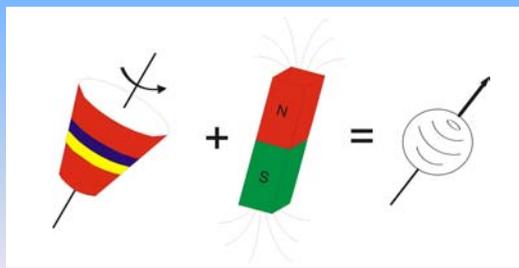
1. NMR basics (4 - 16)
2. Fourier transformation (17 - 21)
3. Quadrature detection (22 - 34)
4. Reduced dimensionality (35 - 37)
5. GFT-NMR (38 - 49)



Peter Schmieder

27.05.2004

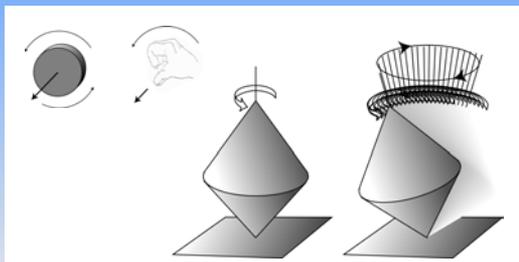
Basis for NMR spectroscopy is the nuclear spin  
that can be viewed as a combination  
of magnet and spinning top



Peter Schmieder

27.05.2004

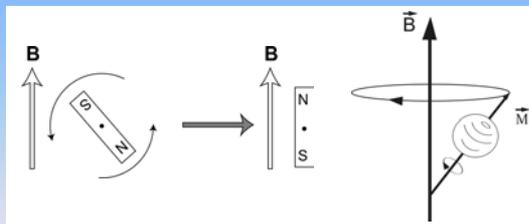
A spinning top has an angular momentum  
the axis of the angular momentum constant in space



Peter Schmieder

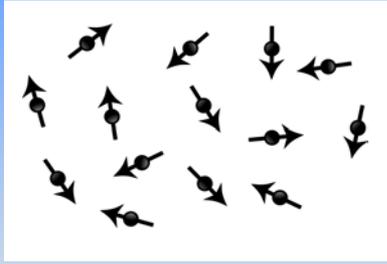
27.05.2004

A magnet orients in the direction of the magnetic field,  
this is prevented by the fact that its a spinning top  
A precession begins



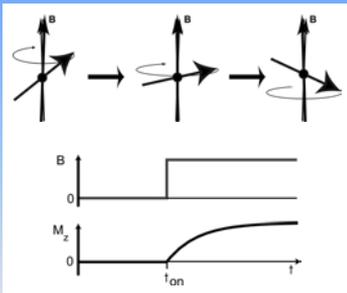
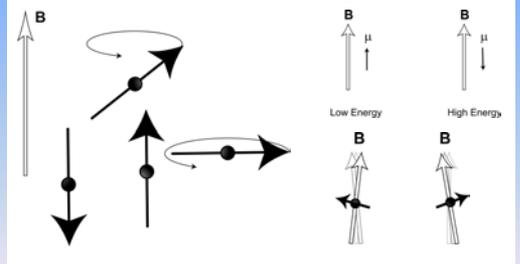
Peter Schmieder

27.05.2004



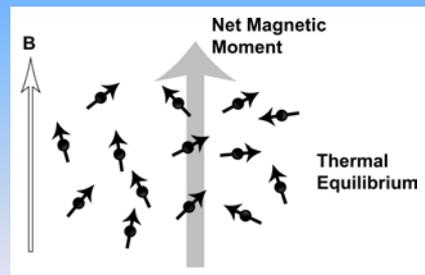
Without an external magnetic field spins are oriented randomly in all possible directions

When a magnetic field is switched on, this does not change immediately, but thermal motion drives spins to orient predominantly along the field

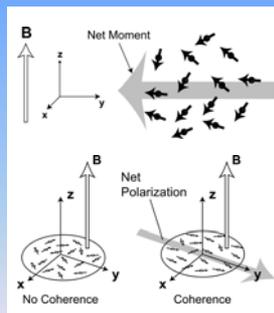
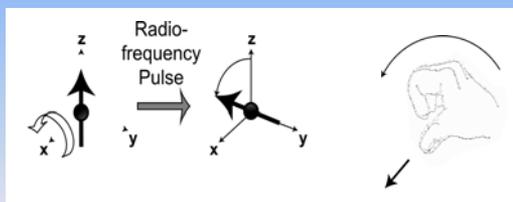


That builds up a net magnetization...

...along the main field, resulting in a Boltzmann distribution

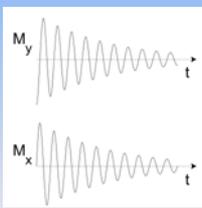
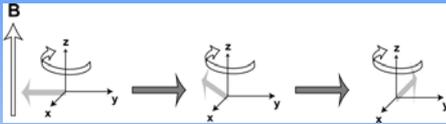


A radio frequency pulse turns the spins....

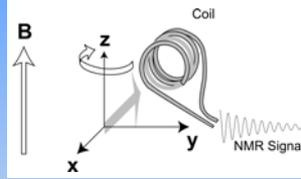


And with them the net magnetization

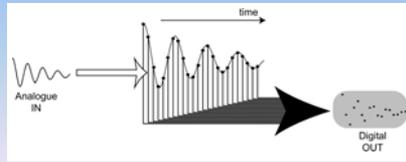
Note the difference between an orientation in the x,y plane and a coherence



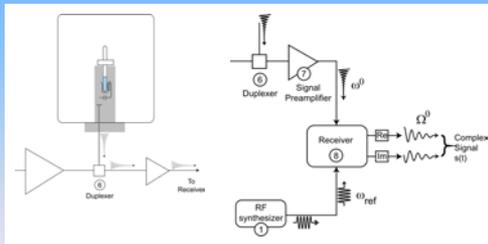
The net magnetization performs a precession  
 $M_y = \cos \omega t \exp(-t/T_2)$   
 $M_x = \sin \omega t \exp(-t/T_2)$   
 $M = M_y + i M_x$   
 $M = \exp(i\omega t) \exp(-t/T_2)$



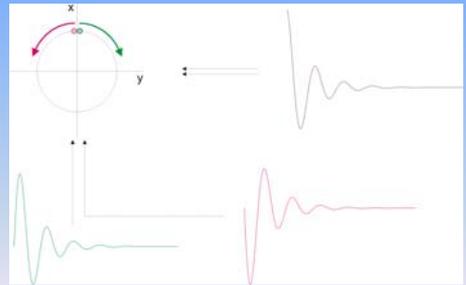
The signal is detected via a coil and then digitized using an ADC (analog-digital-converter)



To make digitization possible, we subtract the original signal from the one returned from the sample

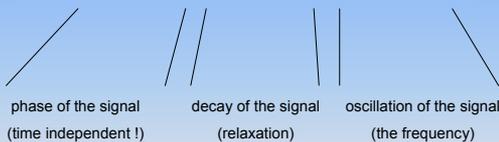


The reference signal is normally the center of the spectrum



A complex NMR signal has the following form

$$s(t) = \exp(i\phi) \exp(-t/T_2) \exp(i\Omega_0 t)$$



phase of the signal  
(time independent !)

decay of the signal  
(relaxation)

oscillation of the signal  
(the frequency)

We ignore the phase factor for the time being and apply a Fourier transformation

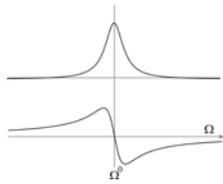
$$s(\Omega) = \int_0^{\infty} s(t) \exp(-i\Omega t) dt$$

$$s(\Omega) = \frac{1}{(1/T_2) - i(\Omega - \Omega_0)}$$

We obtain a complex „Lorentian“ line shape, it consist of an absorptive and a dispersive part

$$A(\Omega) = \frac{(1/T_2)}{(1/T_2)^2 + (\Omega - \Omega_0)^2}$$

$$D(\Omega) = \frac{(\Omega - \Omega_0)}{(1/T_2)^2 + (\Omega - \Omega_0)^2}$$



Now we reintroduce the phase factor

$$S(\Omega) = [A(\Omega) + i D(\Omega)] \exp(i\phi)$$

$$S(\Omega) = R(\Omega) + i I(\Omega)$$

Real and imaginary part are mixtures of absorptive and dispersive line shapes

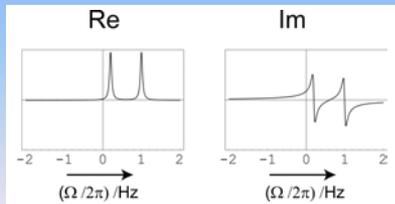
$$R(\Omega) = A(\Omega) \cos \phi - D(\Omega) \sin \phi$$

$$I(\Omega) = D(\Omega) \cos \phi + A(\Omega) \sin \phi$$

We apply a zero-order „phase correction“

$$A(\Omega) = R(\Omega) \cos \phi + I(\Omega) \sin \phi$$

$$D(\Omega) = I(\Omega) \cos \phi - R(\Omega) \sin \phi$$



What if we have only one of the components either cosine or sine

$$\exp(i\alpha) = \cos\alpha + i \sin\alpha$$

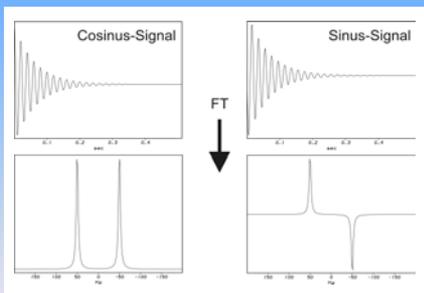
$$\exp(-i\alpha) = \cos\alpha - i \sin\alpha$$

$$\cos\alpha = \frac{\exp(i\alpha) + \exp(-i\alpha)}{2}$$

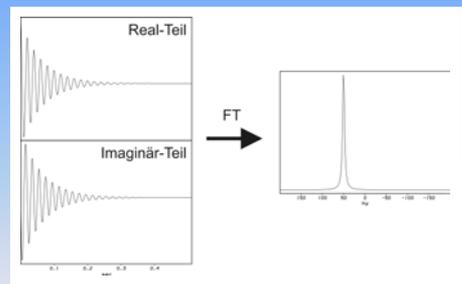
$$\sin\alpha = \frac{\exp(i\alpha) - \exp(-i\alpha)}{2i}$$

(the „famous“ Euler formulas)

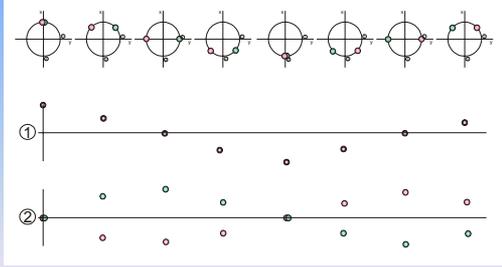
We get two signals each



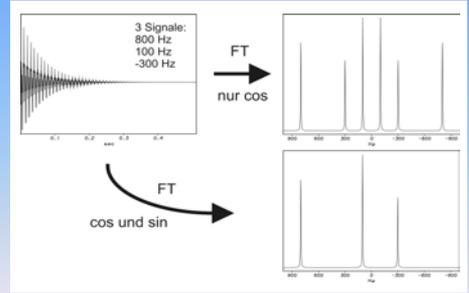
So we have to do a „quadrature detection“



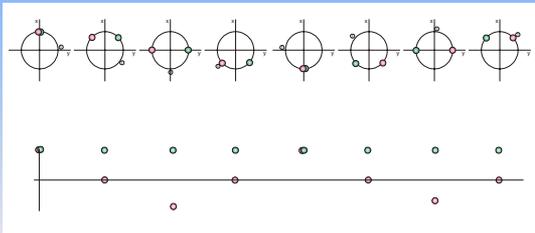
The first way to do it is „States“  
(or Ruben-States-Haberkm)



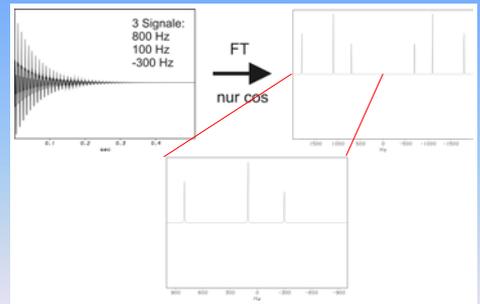
Cosine and sine signal are then combined



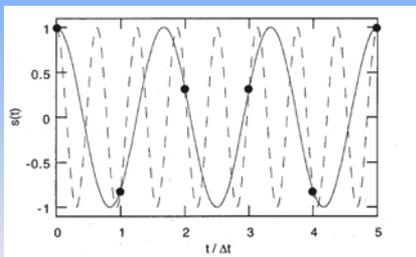
A second way to do it is TPPI  
(time proportional phase increment)



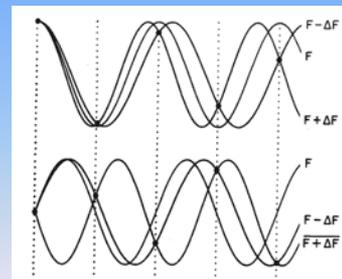
One half of the spectrum is then discarded



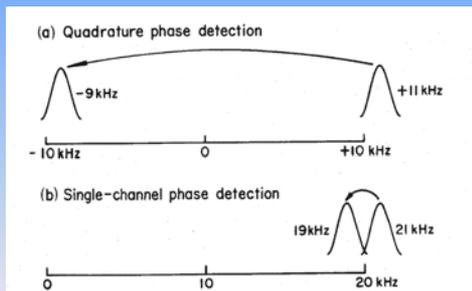
When a continuous signal  $s(t)$  is sampled at evenly spaced intervals as  $s(k\Delta t)$ , the highest detectable frequency is the Nyquist frequency:  $f_n = 1/(2\Delta t)$



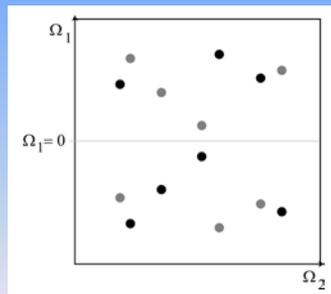
That has consequences depending on the type of quadrature detection



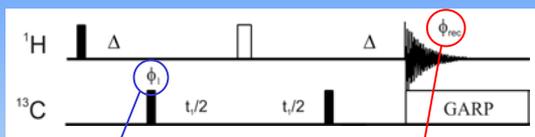
### Aliasing and folding



### The same problem exists for indirect dimensions



### Quadrature detection in F1



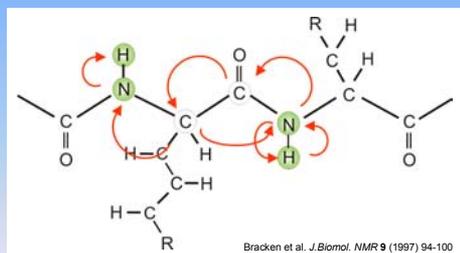
1.FID:  $\phi_1 = x, -x$   
2.FID:  $\phi_1 = y, -y$

1.FID:  $\phi_{rec} = +, -$   
2.FID:  $\phi_{rec} = +, -$

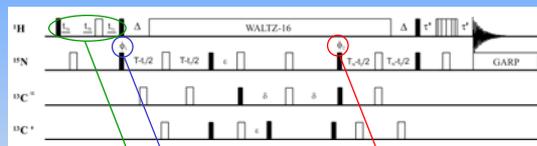
### Methods for quadrature detection

Methode	Phase des Präparationspulses	Empfängerphase	Art der Fouriertransformation	Position der Axialpeaks
Redfield	$x(t_1 + 0^*\Delta)$	$x$	reell	Zentrum
	$y(t_1 + 1^*\Delta)$	$x$		
	$x(t_1 + 2^*\Delta)$	$-x$		
TPPI	$y(t_1 + 3^*\Delta)$	$-x$	reell	Rand
	$x(t_1 + 0^*\Delta)$	$x$		
	$y(t_1 + 1^*\Delta)$	$x$		
SHR	$-x(t_1 + 2^*\Delta)$	$x$	komplex	Zentrum
	$x(t_1 + 3^*\Delta)$	$x$		
	$y(t_1 + 0^*\Delta)$	$x$		
TPPI-States	$x(t_1 + 2^*\Delta)$	$x$	komplex	Rand
	$y(t_1 + 0^*\Delta)$	$x$		
	$-x(t_1 + 2^*\Delta)$	$-x$		
	$-y(t_1 + 2^*\Delta)$	$-x$		

### (H)N(COCA)NH and HN(COCA)NH



### (H)N(COCA)NH and HN(COCA)NH

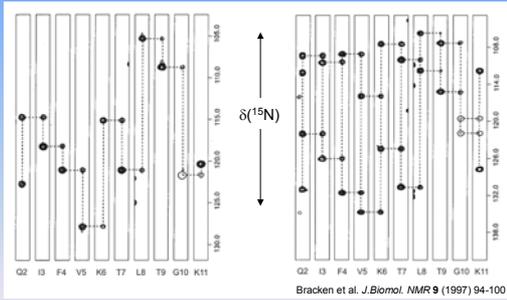


quadrature detection in  $t_1$       quadrature detection in  $t_2$

reduced dimensionality: evolution on  $^1\text{H}^N$  is recorded concomitantly with the evolution on N but without any scheme for quadrature detection

(H)N(COCA)NH

HN(COCA)NH



Peter Schmierer

27.05.2004

reduced dimensionality:

nD information in an (n-1)D spectrum

GFT-NMR:

Generalization of the this principle

nD information in an (n-k)D spectrum

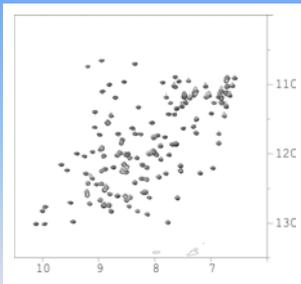


Peter Schmierer

27.05.2004

First the "target dimensionality" is selected

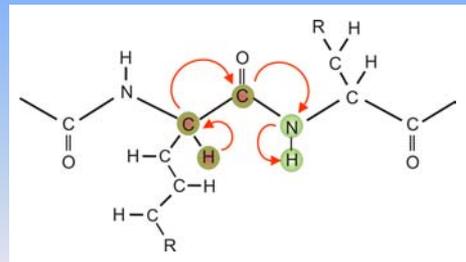
If there is barely overlap in the <sup>15</sup>N-HSQC the target dimensionality is 2D



Peter Schmierer

27.05.2004

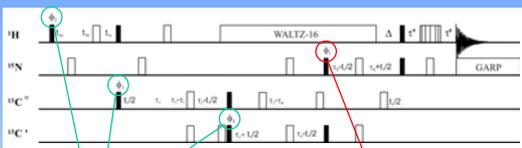
5D-HACACONHN and (5,2)D-HACACONHN



Peter Schmierer

27.05.2004

(5,2)D HACACONHN



"normal" quadrature detection

"special" modulation scheme, 16 spectra are recorded



Peter Schmierer

27.05.2004

Three additional dimensions require 8 "basic spectra"

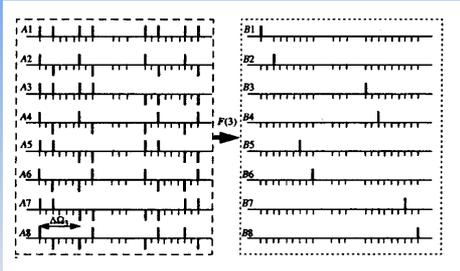
- S1  $\propto \cos(\Omega_0 t) \cos(\Omega_1 t) \cos(\Omega_2 t) \cos(\Omega_3 t)$
- S2  $\propto \cos(\Omega_0 t) \sin(\Omega_1 t) \cos(\Omega_2 t) \cos(\Omega_3 t)$
- S3  $\propto \cos(\Omega_0 t) \cos(\Omega_1 t) \sin(\Omega_2 t) \cos(\Omega_3 t)$
- S4  $\propto \cos(\Omega_0 t) \sin(\Omega_1 t) \sin(\Omega_2 t) \cos(\Omega_3 t)$
- S5  $\propto \cos(\Omega_0 t) \cos(\Omega_1 t) \cos(\Omega_2 t) \sin(\Omega_3 t)$
- S6  $\propto \cos(\Omega_0 t) \sin(\Omega_1 t) \cos(\Omega_2 t) \sin(\Omega_3 t)$
- S7  $\propto \cos(\Omega_0 t) \cos(\Omega_1 t) \sin(\Omega_2 t) \sin(\Omega_3 t)$
- S8  $\propto \cos(\Omega_0 t) \sin(\Omega_1 t) \sin(\Omega_2 t) \sin(\Omega_3 t)$



Peter Schmierer

27.05.2004

Proper combination results in spectra with only one peak

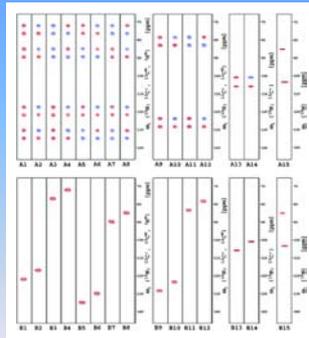
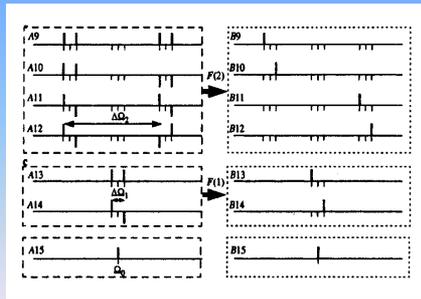


FT of S1-S8

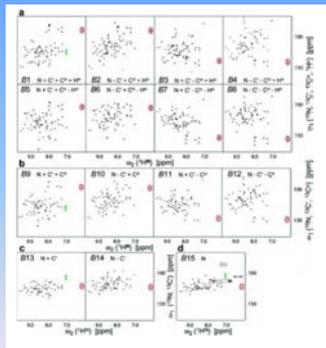
That corresponds to a matrix operation

$$\begin{bmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \\ B6 \\ B7 \\ B8 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \\ A7 \\ A8 \end{bmatrix}$$

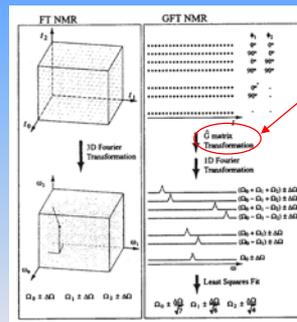
It is difficult to decide which peak is which, therefore "central peaks" are recorded



That's how it looks



The result are 15 2Ds



The G-matrix does it all in the time domain

## Comparison of numbers:

## (5,2)D HACACONHN

15 spectra can be recorded in 34 min and yields a set of two-dimensional spectra

## 5D HACACONHN

$10^*(t_1) \times 11^*(t_2) \times 22^*(t_3) \times 13^*(t_4) \times 512^*(t_5)$

1 scan per FID, 1 second per scan 5.83 days

Processed to the same resolution 618 GB



Peter Schmieder

27.05.2004

## The acquisition of multidimensional NMR spectra within a single scan

Lucio Frydman<sup>1</sup>, Iuli Scheer<sup>1</sup>, and Adonis Lupulescu

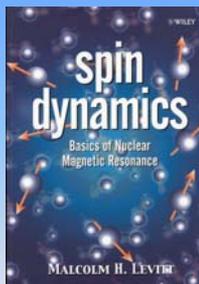
*Proc. Natl. Acad. Sci. USA* **99**, 15858-15862 (2002)

4D spectrum in 194 msec.....



Peter Schmieder

27.05.2004



Peter Schmieder

27.05.2004