Magnetic Resonance Imaging

II. MRI Technology

A. Limitations to Spatial Information

B. Localized Information

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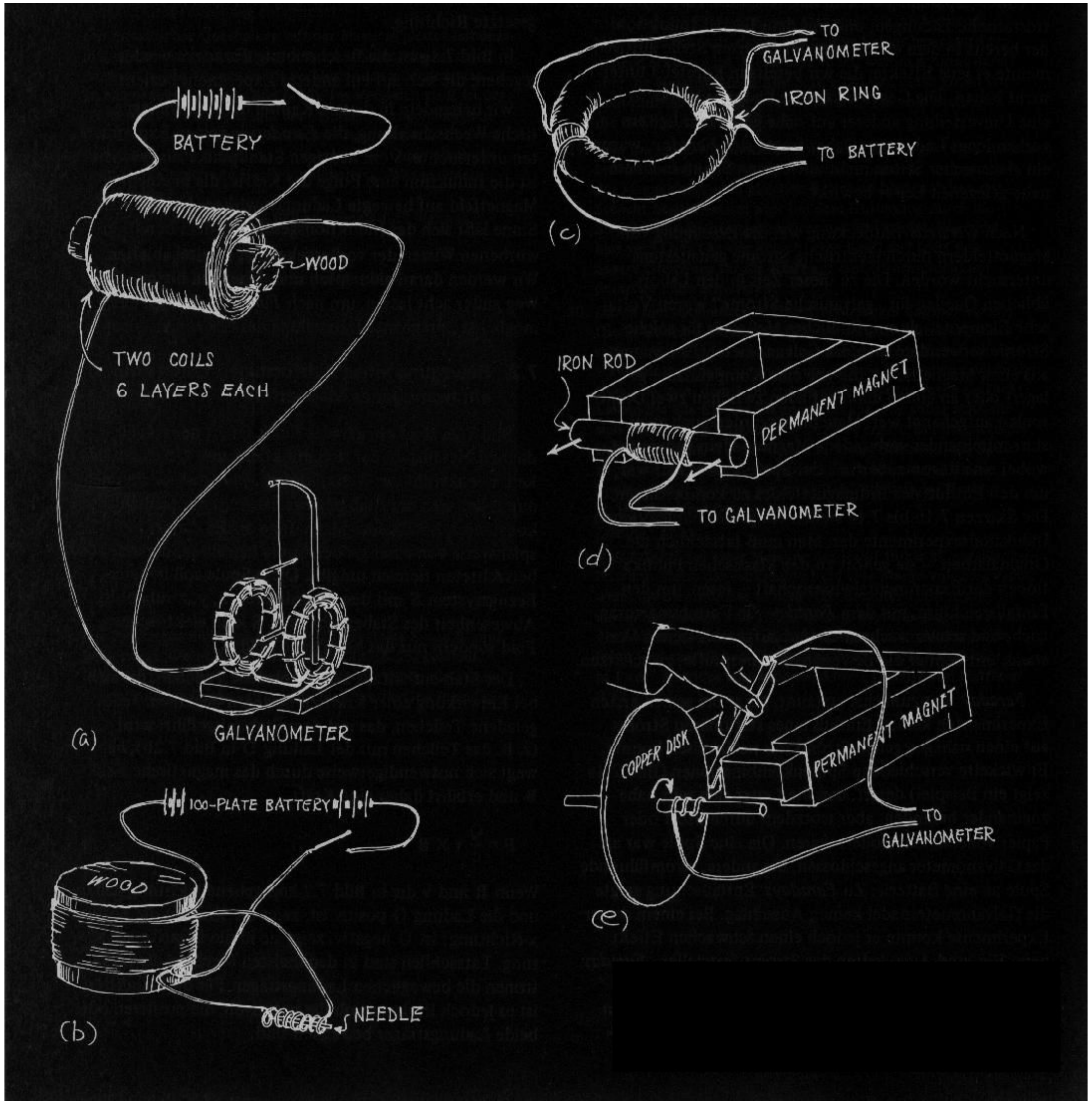
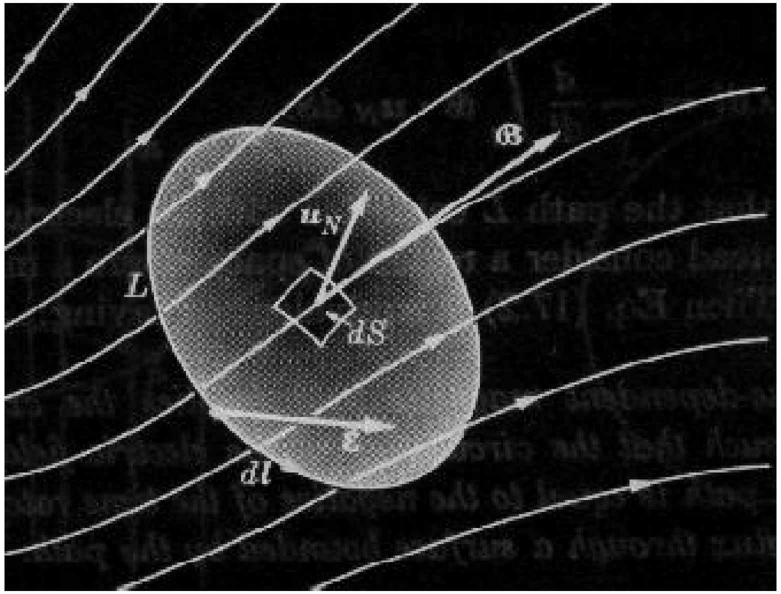
A. Limitations to Spatial Information



Michael Faraday
"Experimental Researches in Electricity", London 1839

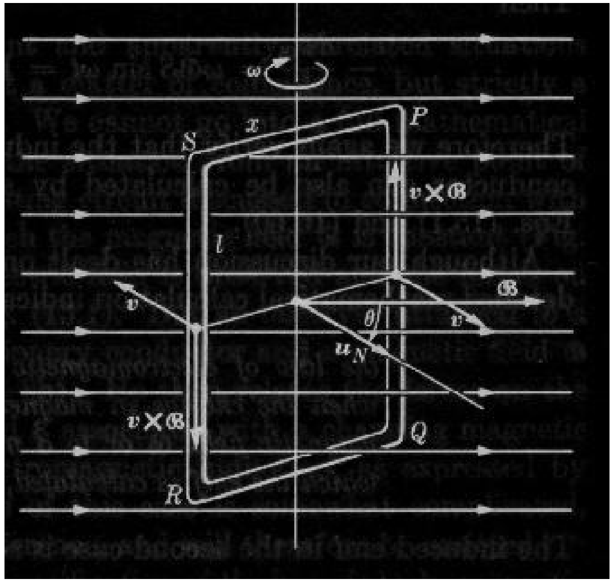
Electromagnetic Induction



$$\oint_L \vec{E}\, d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\, d\vec{S}$$

electromotive force

rate of change of magnetic flux

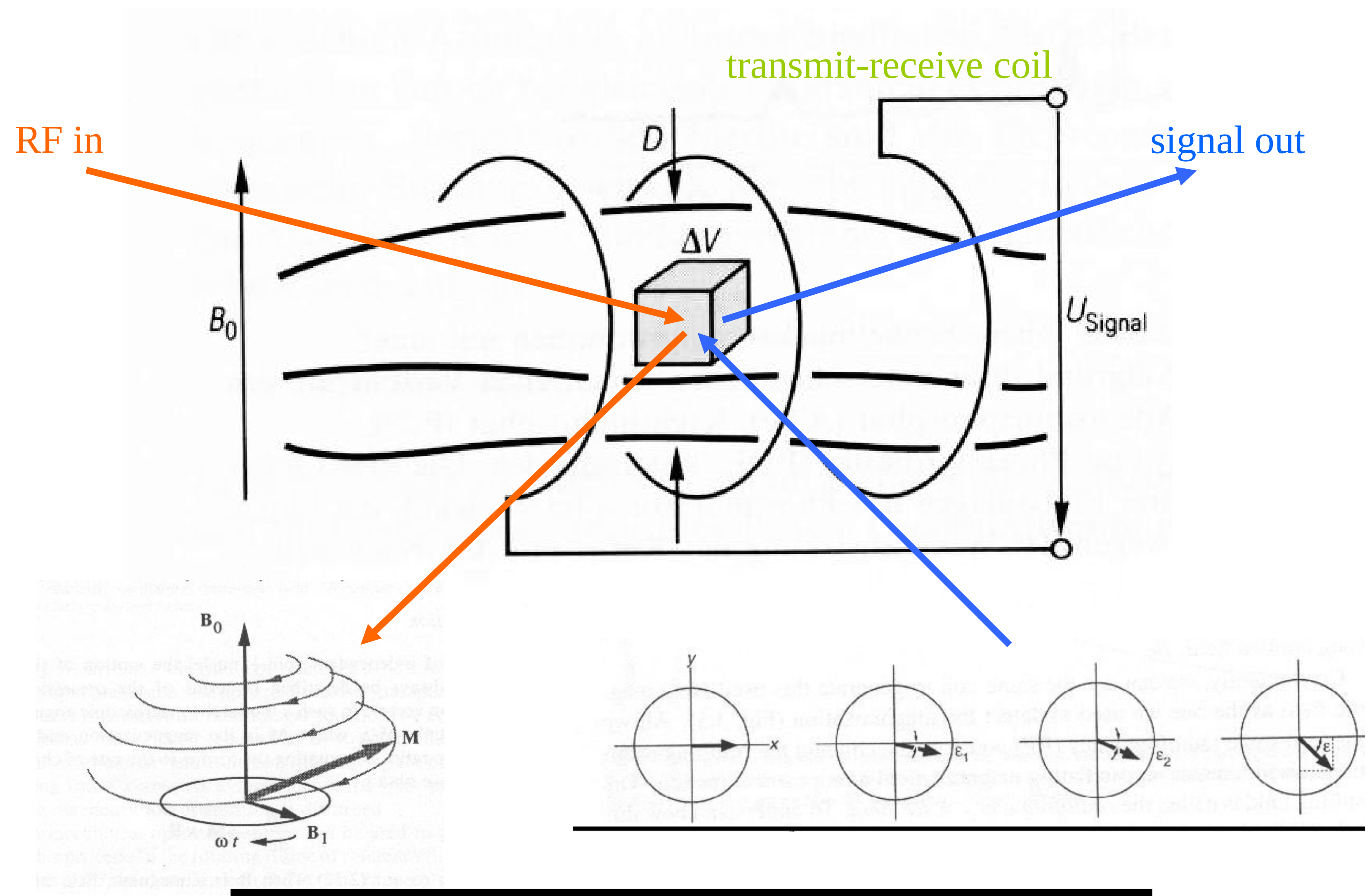


$$U_{ind} = -\frac{d\Phi}{dt}$$

negative sign indicates “resistance” to change

Lenz’s Rule

Basic Experiment



What signal do we get from the voxel ?

Principle of Reciprocity, Signal Intensity

The emf induced in a RF probe by magnetic moment M from a voxel ΔV at a specific position is determined by the magnetic field amplitude B_1 at that position when unit current flows through the coil,

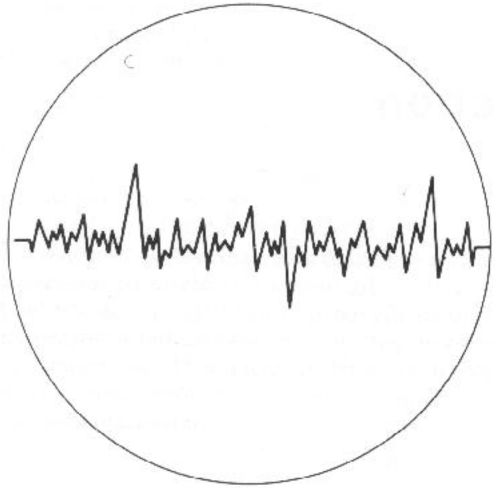
$$\Delta U_{ind} = -\frac{d}{dt}\left(\vec{B}_1 \cdot \vec{M}\right)$$

$$\vec{M} = M_0 \begin{pmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{pmatrix} \; ; \; \vec{B}_1 = \begin{pmatrix} B_{1X} \\ B_{1Y} \\ B_{1Z} \end{pmatrix} \; ; \; B_{1X} = B_{1Y} = B_{tr}$$

maximum signal amplitude $\Delta U_{ind\,max} \approx \omega M_0 B_{tr}$

Noise

Noise are stochastic fluctuations of induced voltage generated by thermal motion of charges and currents



Nyquist, mean quadratic voltage $$\langle U_n^2 \rangle = 4kfTR$$

$$TR = \underbrace{T_C R_C}_{\text{coil}} + \underbrace{T_S R_S}_{\text{sample/body}}$$

coil sample/body

k, Boltzmann's constant
f, spectral bandwidth
T_C, coil temperatur
R_C, coil resistance
T_S, sample/body temperature
R_S, sample/body resistance

signal/noise from ΔV

$$\frac{U_{ind\,max}}{\sqrt{\langle U_n^2 \rangle}} \equiv \frac{S}{N} \approx \frac{\omega M_0 B_{tr}}{\sqrt{4kf\left(T_C R_C + T_S R_S\right)}}$$

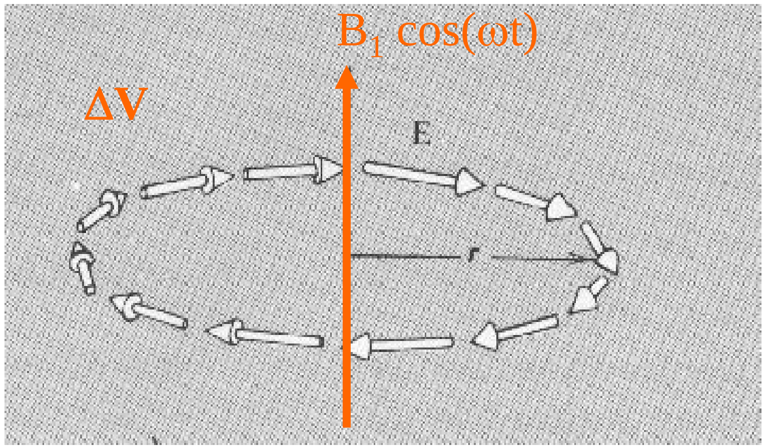
Sample / Body Resistance

Sample in transmit-receive circuit causes enery loss "Joule heating"

"resistance=mean powerdissipation / (mean current)^2"

Unit ac of frequency ω generates B_1 which induces E in sample/body of conductivity σ

$$R_S = \frac{1}{2} \int_{\text{sample}} \sigma E^2 dV$$

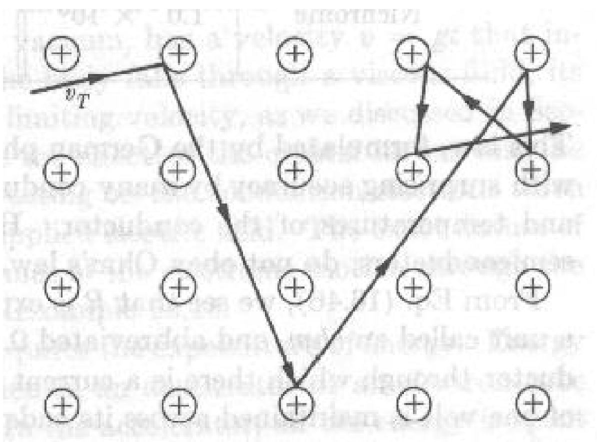


B_1 homogeneous in small voxel $\vec{E} = \frac{\omega}{2} \vec{r} \times \vec{B}_1$

$$R_S = \frac{1}{4} \sigma \omega^2 \underbrace{\int_{\text{sample}} \left(\vec{r} \times \vec{B}_1 \right)^2 dV}_{\theta_S}$$

geometry factor for coil and object

Coil Resistance



hindered motion of electrons in lattice of the conductor
confined to skin on the surface of the coil ("skin – effect")

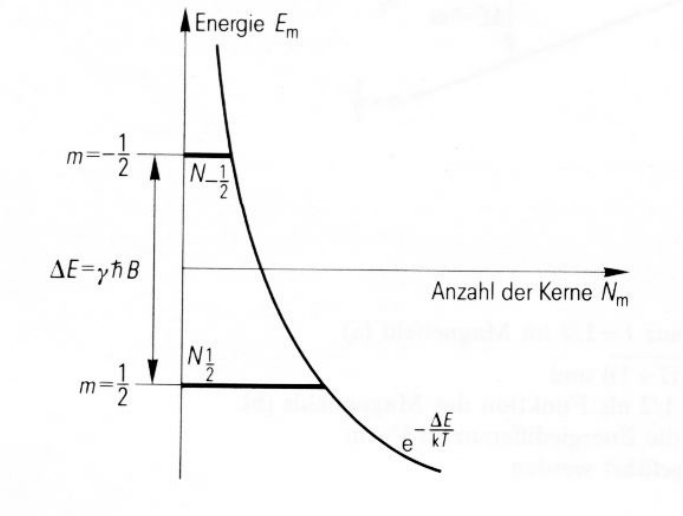
effective coil resistance
single layer solenoid

$$R_C = \sqrt{\omega\mu_r\mu_0\rho(T_C)}\underbrace{\frac{L}{\sqrt{2P}}\eta}_{\theta_C}$$

geometry factor for coil

L, conducter length
P, conductor circumfence
$\mu_r\mu_0$, permeability
$\rho(T_C)$, resistivity
$\eta=5$, correction to skin-effect for tight winding

Thermal Equilibrium Magnetization, total S/N



Boltzmann distribution of population of energy levels generates excess magnetization in thermal equilibrium

$$M_0 = N\gamma^2\hbar^2 I(I+1)B_0 / 3kT_S$$

"Curie law" for ambient T_S

- N, number of spins in ΔV
- I, spin quantum number
- $\hbar$, Planck constant
- γ, gyromagnetic ratio
- B_0, Zeeman field strength

Total S/N from voxel ΔV

$$\frac{S}{N} \approx \frac{\omega^2 N\gamma\hbar^2 I(I+1)}{\sqrt{T_C T_S^2 \sqrt{\omega\mu_r\mu_0\rho}\,\theta_C + T_S^3 \frac{1}{4}\sigma\omega^2\theta_S}} \frac{B_{tr}}{6\sqrt{k^3 f}}$$

S/N in NMR and MRI

Standard conditions (room temperature, 1 Tesla,
1 Liter physiological saline solution, solenoid, …)

^{1}H, S/N~1: N~5 10^{18}
1 mm^3 water: N=6.7 10^{18}

in vitro NMR

small sample, ~cm^3: $R_C >> R_S$

$$\frac{S}{N} \sim \frac{\omega^{7/4} N \gamma I(I+1)}{\sqrt{T_C T_S}}$$

- S/N increases fast with Larmor frequency or ***high Zeeman*** field
- ***Cooling electronics*** helps
- Signal averaging, heteronuclear labelling, …

in vivo MRI

large object: $R_C << R_S$

$$\frac{S}{N} \sim \frac{\omega N \gamma I(I+1)}{T_S^{3/2}}$$

- ***only water*** can be reasonably measured
- ***Resolution limited*** to ~ 1 mm^3
- Restricted experiment time, no labelling possible, …

Hyperpolarization

Non-thermal initial polarization may lift the limitations of MRI, $S/N \sim M_0$

Dynamic nuclear polarization

$\gamma_e / \gamma_{1H} \approx 660$, $T_p \approx 1.1\,K$ Curie law !

Parahydrogen induced polarization

$$\frac{1}{\sqrt{2}}\left(\langle + | \langle - | - | - \rangle | + \rangle\right)$$ H_2 in singlet state 100%

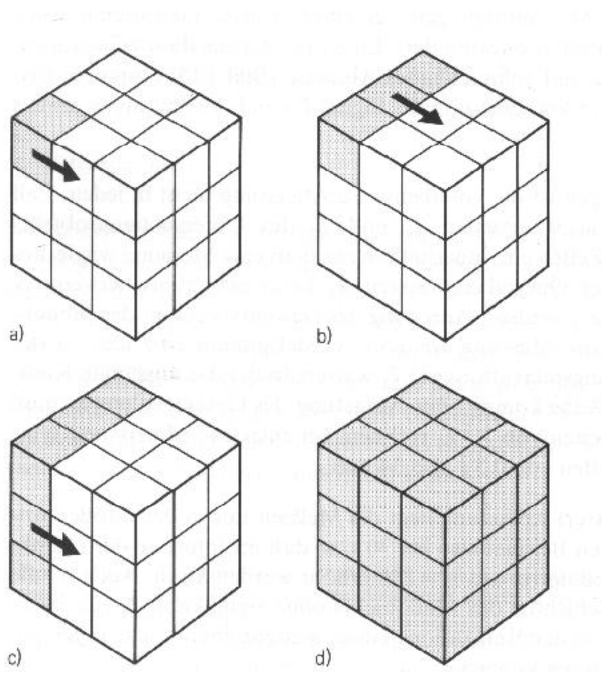
Laser polarization

Nuclear spins of ^{129}Xe, ^{3}He gain excess magnetization ~20% by collisions with optically excited Ions

B. Localized Information

- surface coil: high S/N (small θ_S), inhomogeneous, poor localization

- relaxation behavior:



N^3, number of voxels
T_1, longitudinal relaxation time
T_2, (T_2^*), transverse (effective) relaxation time
τ, duration of experiment

individual sampling $\tau \sim N^3\, T_1 \sim 36$ h at $T_1 = 0.5$ s, N=64

line wise sampling $\tau \sim N^2\, T_1 \sim 1/2$h

plane wise sampling $\tau \sim N\, T_1 \sim 1/2$min
or $\sim$ 1/2h at resolution N=256

Selective excitation of the magnetization in a plane of the object.

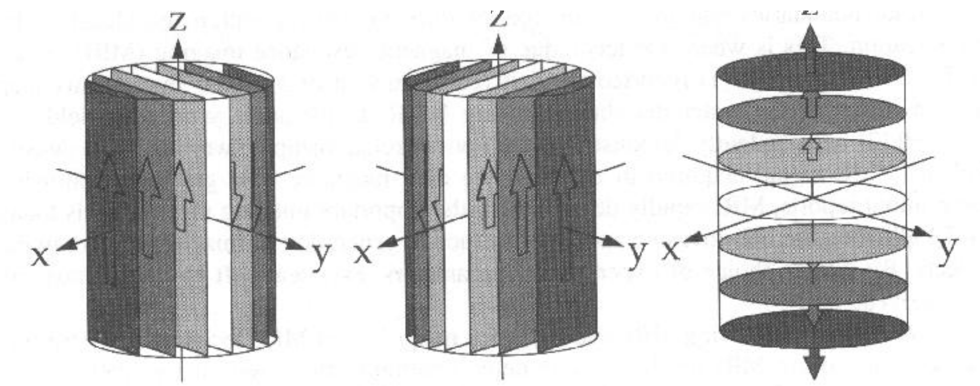
Zeeman Field Gradients

Field changes linearly with position

$$\vec{B}(\vec{r}) = \vec{B}_0 + \vec{G} \cdot \vec{r}$$

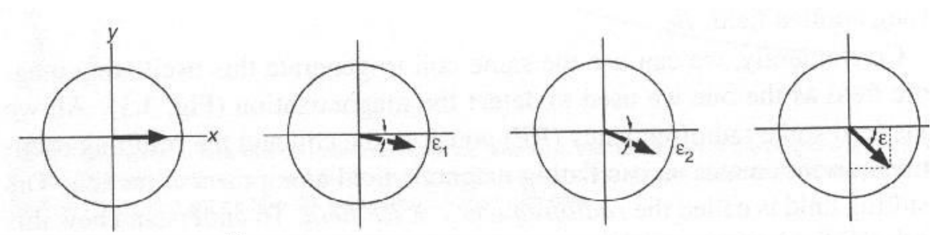
Larmor frequency changes accordingly

$$\vec{\omega}(\vec{r}) = \gamma\vec{B}_0 + \gamma\vec{G} \cdot \vec{r}$$



x-gradient *y-gradient* *z-gradient*

Local precession by gradient offset
in rotating frame:



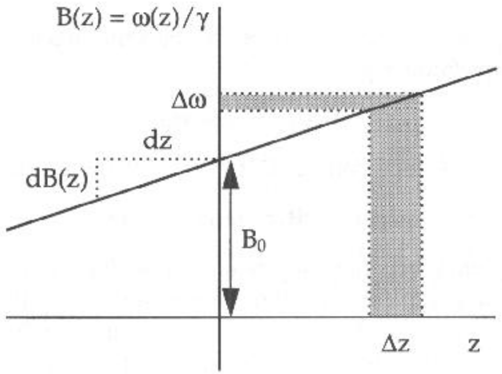
"k-space"

$$\vec{k}(t) = \gamma\int_0^t \vec{G}(t')dt'$$

$$M_\perp(\vec{r}, t) = M_\perp(\vec{r}, 0)e^{-i\vec{k}(t)\cdot\vec{r}}$$

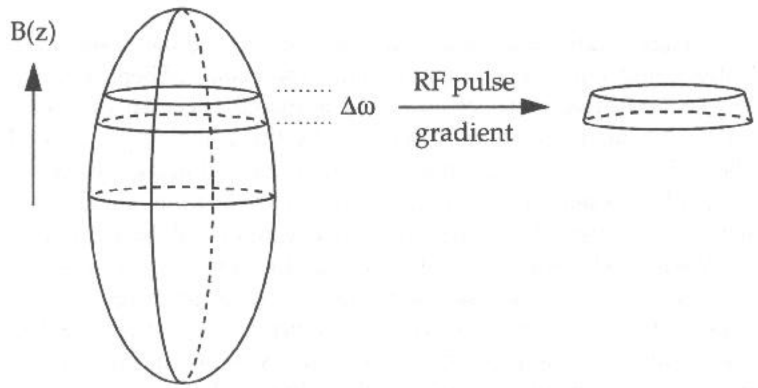
Linear correlation of
resonance/precession frequency and spatial position

Selective Excitation



Transverse magnetization in rotating frame after pulse of length τ, amplitude B_1 along x-axis in
small flip angle approximation

$$M_\perp(\vec{r},\tau) \approx \gamma M_0(\vec{r}) \int_0^\tau B_1(t) e^{-\gamma \vec{G}\vec{r}t} dt$$



"slice profile" = "FT of pulse shape"

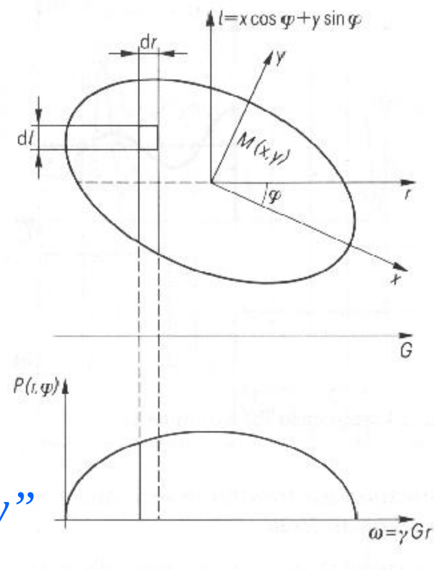
Gradient strength to excite slice of width ΔZ by RF pulse of bandwidth BW

$$G = \frac{BW}{\gamma \Delta Z}$$

Encoding of Spatial Dimension

Gradient along one direction r (1D):

$$M_\perp(t) = \int_{\text{sample}} M_\perp(\vec{r},0)\, e^{-i\vec{k}(t)\cdot\vec{r}}\, dV = \int P(r)\, e^{-ik_r(t)\cdot r}\, dr$$



Projection of Magnetization along r is FT of Signal as function of k.

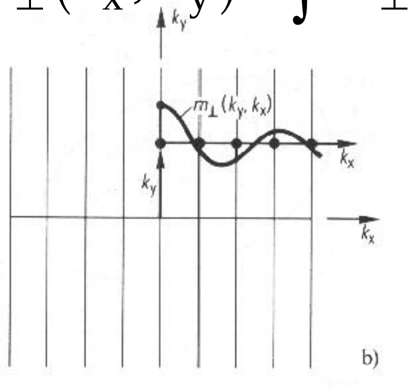
N signal samples at $\Delta k_r = \gamma G_r \Delta t$ and from Nyquist Theorem:

Total spatial dimension resolution: $FOV = 2\pi / \Delta k_r$ *"field of view"*

Spatial resolution (technically): FOV/N

Independent gradients along two directions x, y (2D):

$$M_\perp(k_x, k_y) = \int M_\perp(x,y)\, e^{-ik_x(t)\cdot x - ik_y(t) y}\, dxdy$$



sampling 2D k-space
with gradient-echo sequence

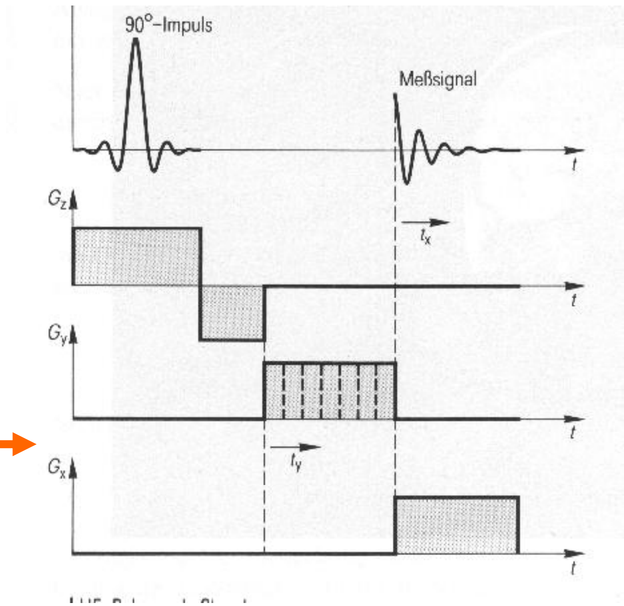
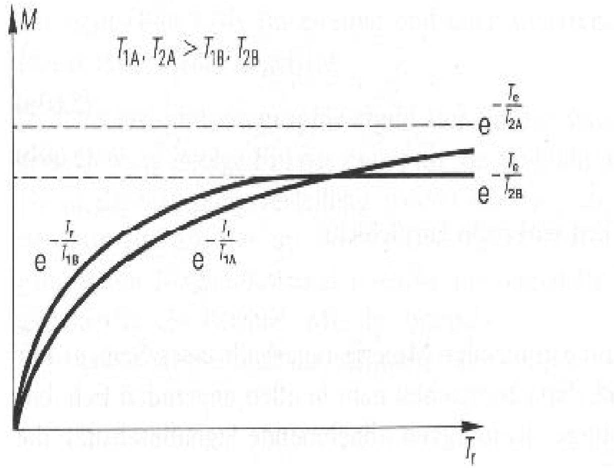
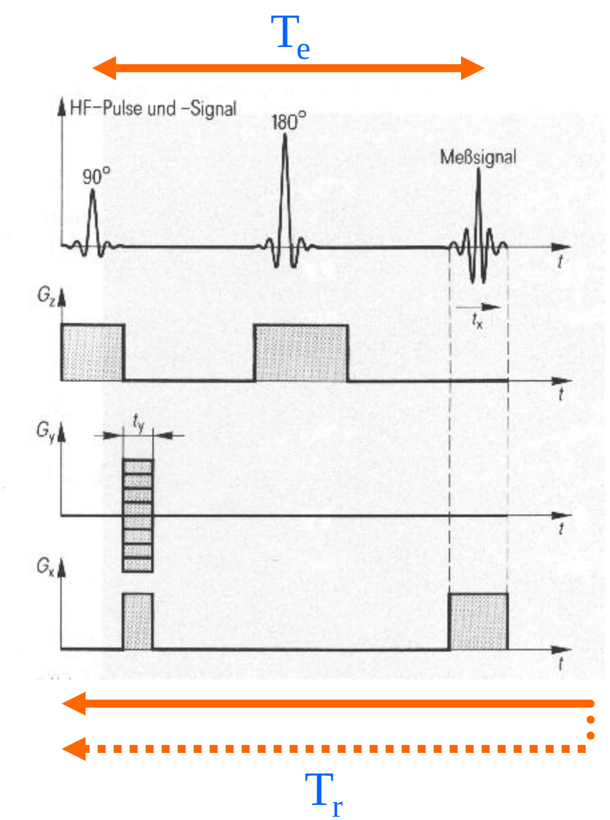


Image Contrast

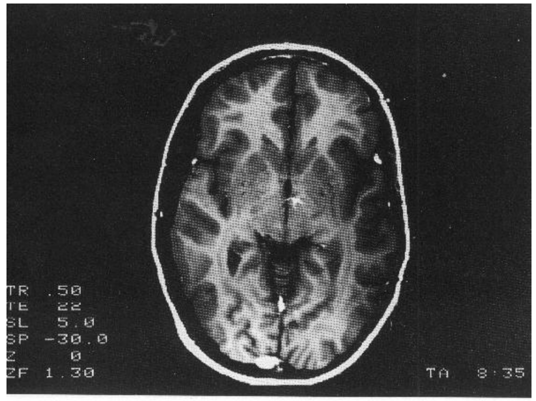
Relaxation times weighted local magnetization

$$M_{\perp}(x,y)=\rho(x,y)e^{-\frac{T_e}{T_2}}\left(1-e^{-\frac{T_r}{T_1}}\right)$$



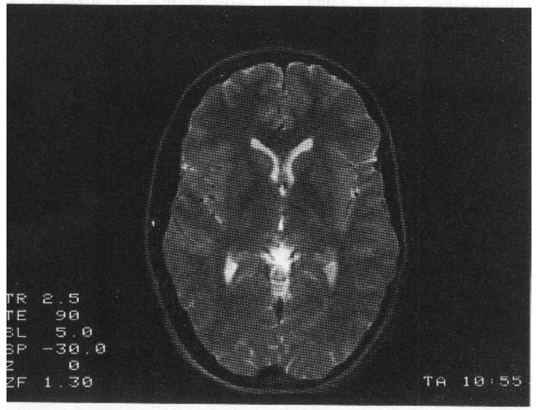


white matter light,
Grey matter dark
T_r = 450 ms



contrast inversion

white matter dark,
grey matter light
T_r = 3.3 s



MR Imaging = localized determination of MR parameters

… which need medical interpretation …

… so, let's practice …