

Fourier Transform and Spectral Analysis

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Introduction

- Historical Background
- Fourier Synthesis
- Fourier Analysis
- Time Frequency Analysis
- Applications

Jean Baptiste Joseph Fourier (1768-1830)

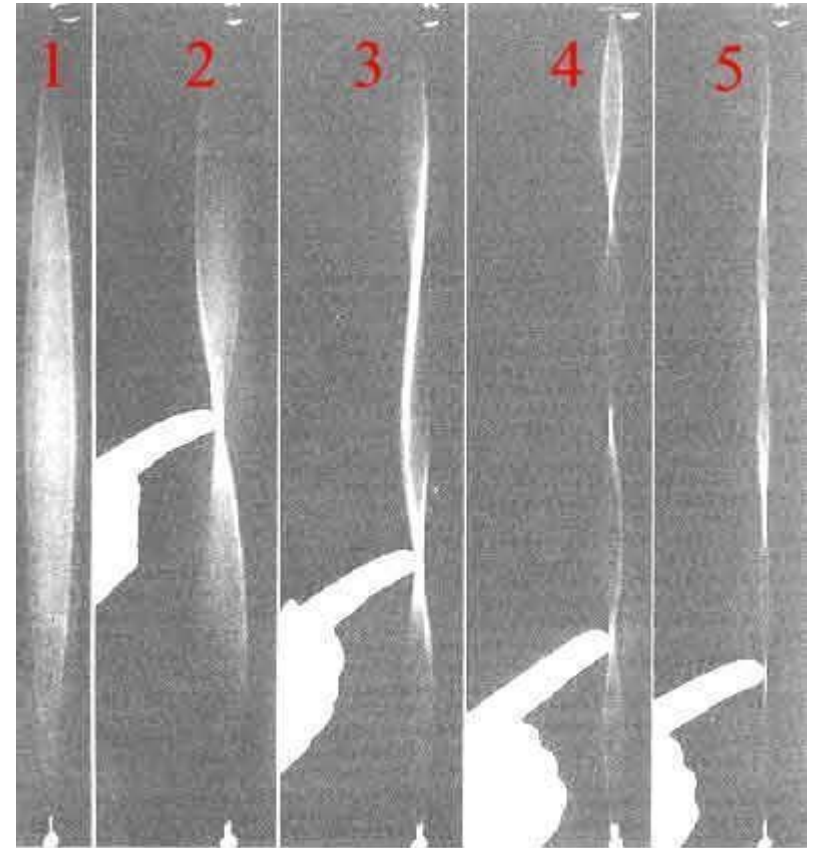
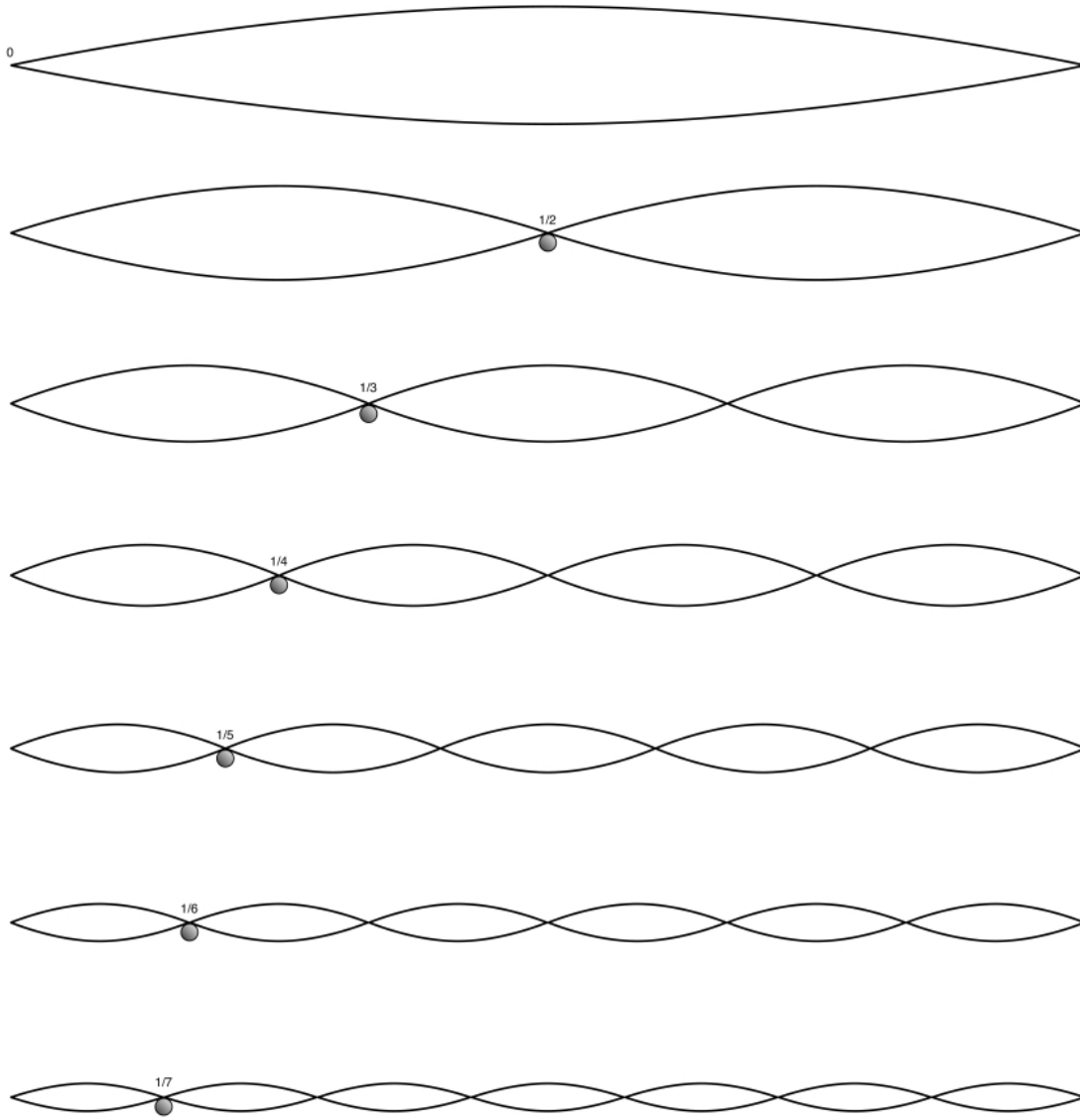


*Théorie analytique
de la chaleur (1822)*

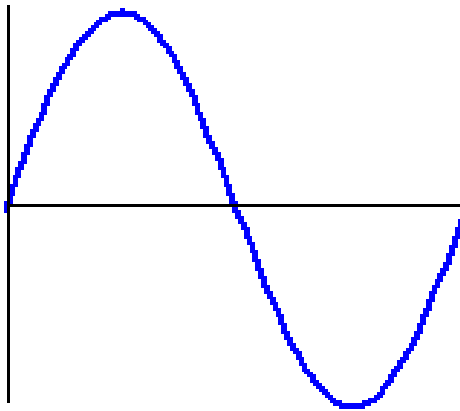
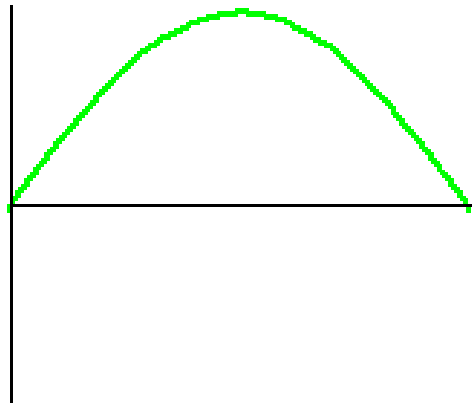
**A continuous periodic
function can be written
as a sum of sine-waves**

$$f(t) = a_0 + \sum_{n=1}^N (a_n \cos(n\omega t) - b_n \sin(n\omega t)).$$

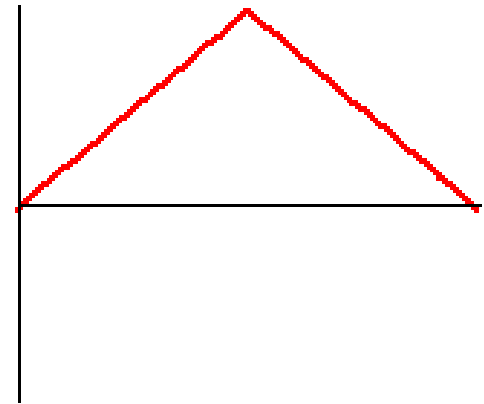
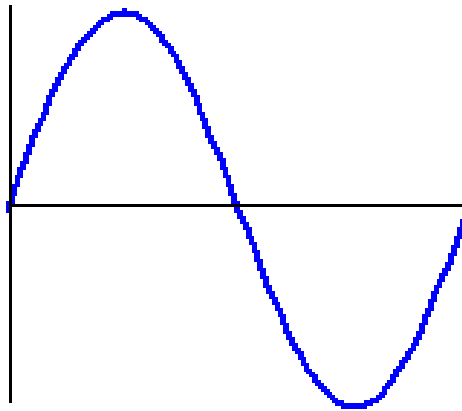
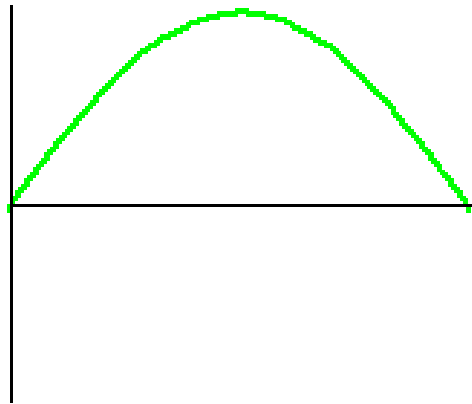
Vibrating Strings



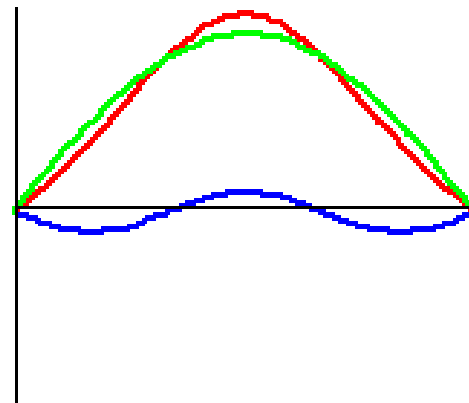
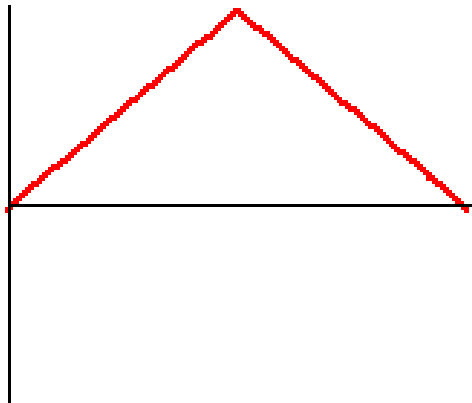
Vibrating Strings

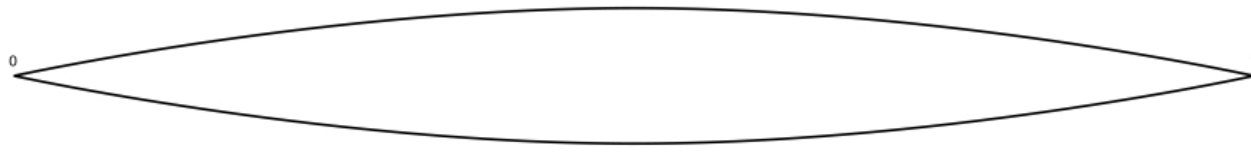


Vibrating Strings

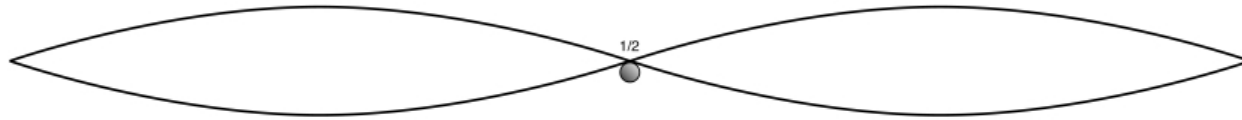


Vibrating Strings

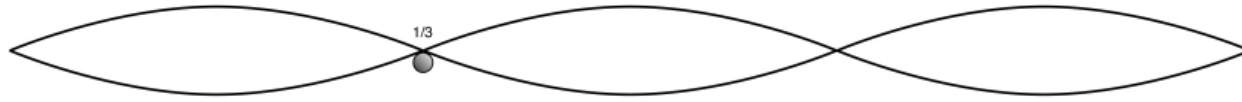




$$a_1 \quad \omega$$



$$a_2 \quad 2\omega$$



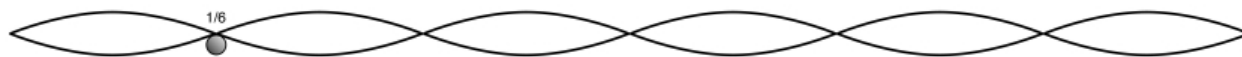
$$a_3 \quad 3\omega$$



$$a_4 \quad 4\omega$$



$$a_5 \quad 5\omega$$

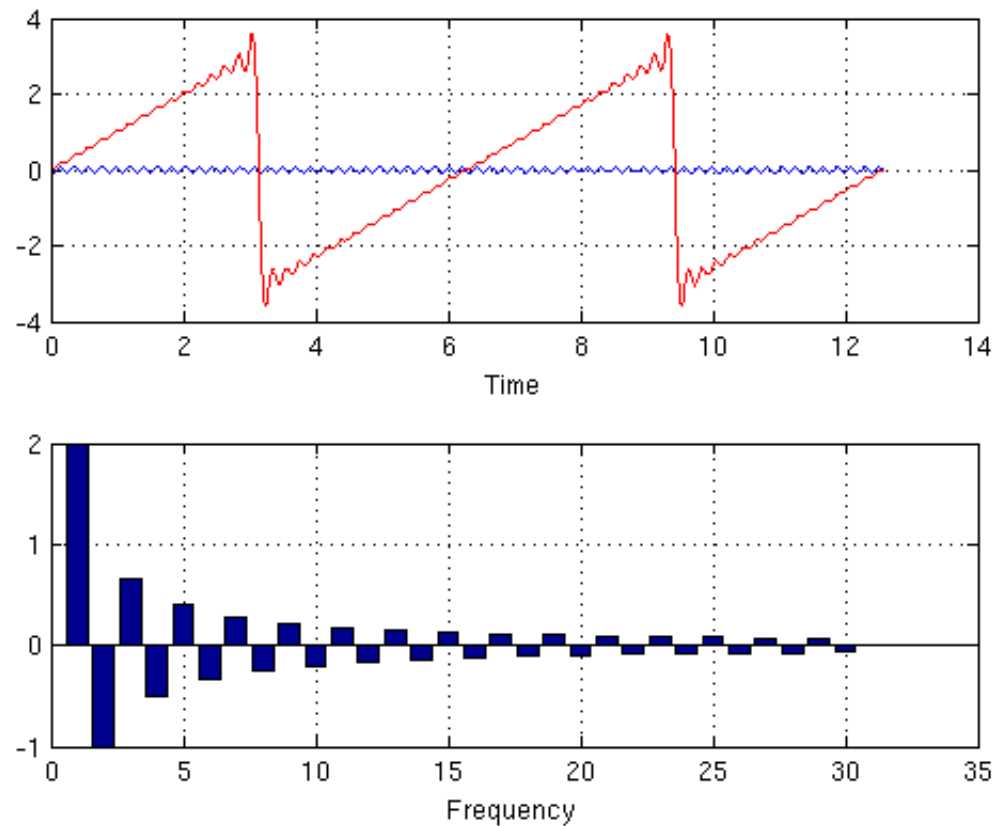


$$a_6 \quad 6\omega$$



$$a_7 \quad 7\omega$$

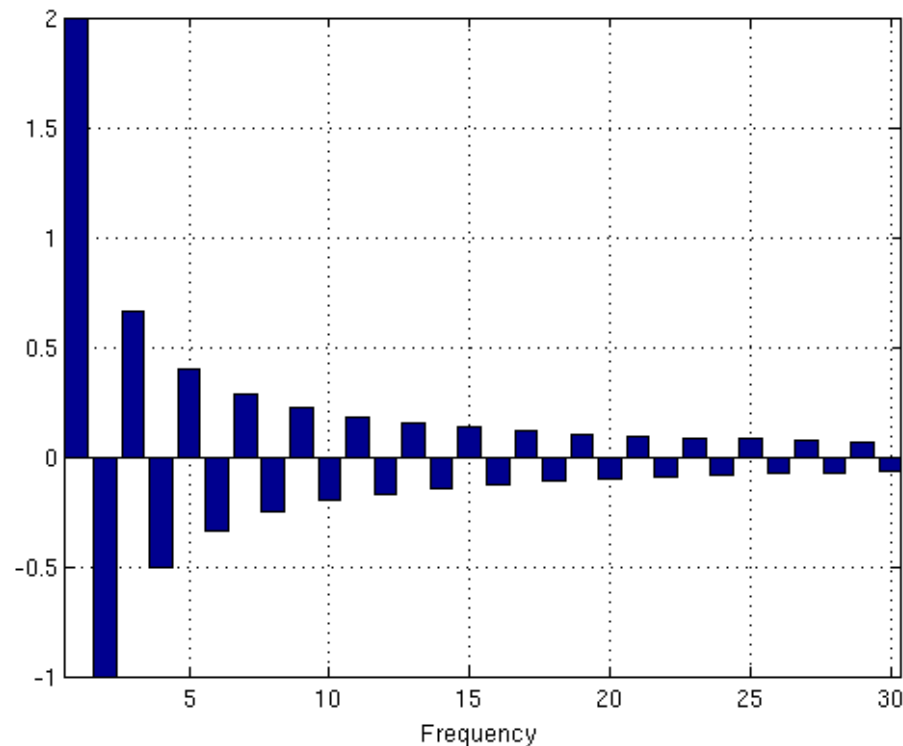
Sawtooth



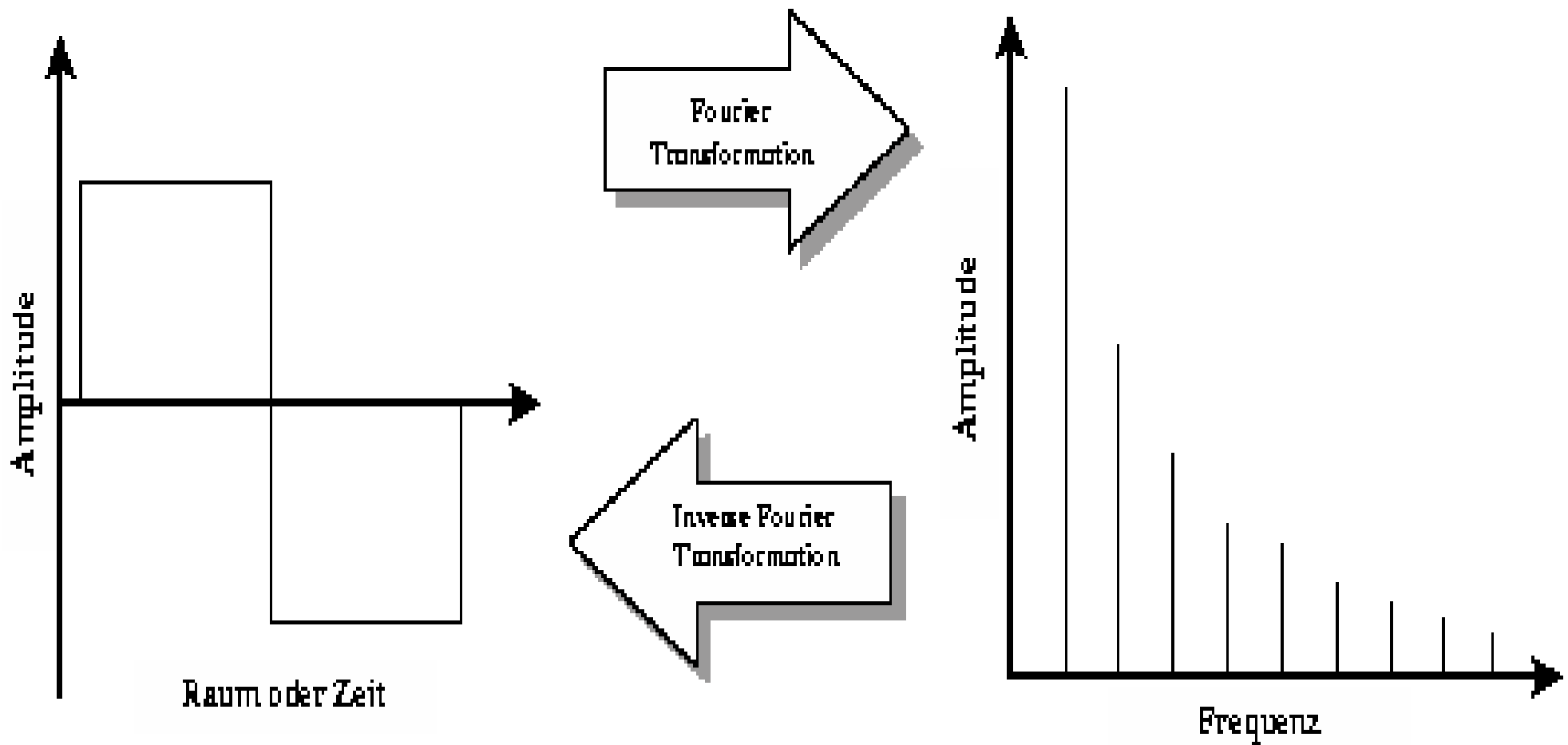
$$c(t) = 2 \sum_{k=1} \frac{(-1)^k}{k} \sin(k \cdot t)$$

A continuous periodic function can be written
as a sum of sine-waves

$$f(t) = a_0 + \sum_{n=1}^N (a_n \cos(n\omega t) - b_n \sin(n\omega t)).$$



Coefficients are forming the spectrum



$$f(t) = a_0 + \sum_{n=1}^N (a_n \cos(n\omega t) - b_n \sin(n\omega t)).$$

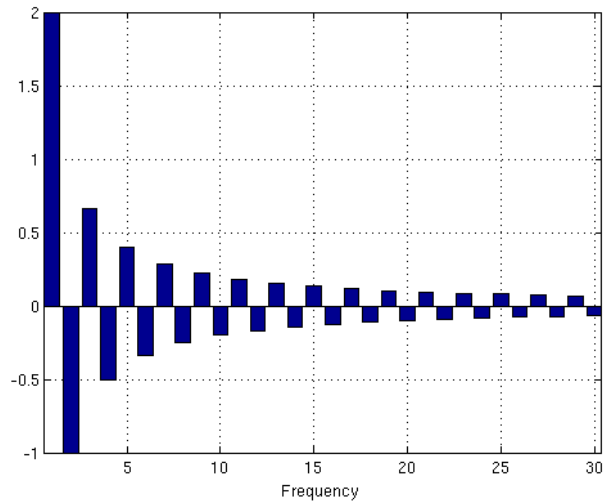
Fourier series and complex numbers

$$f(t) = a_0 + \sum_{n=1}^N (a_n \cos(n\omega t) - b_n \sin(n\omega t)).$$

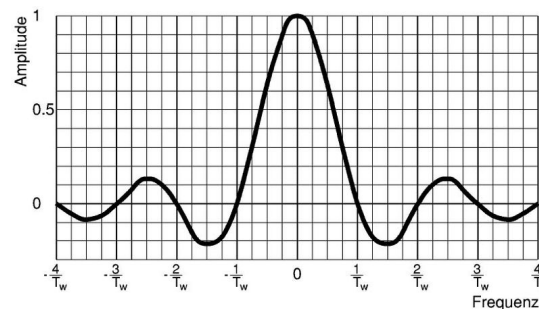
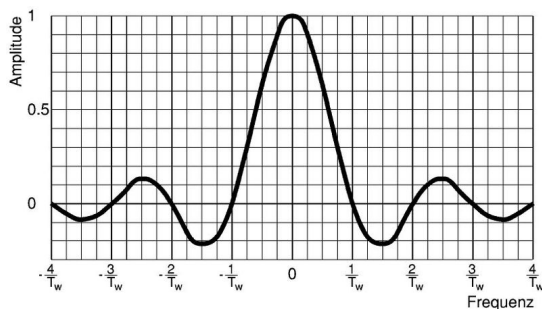
$$= a_0 + \sum_{n=1}^N \frac{1}{2} (a_n(e^{in\omega t} + e^{-in\omega t}) + ib_n(e^{in\omega t} - e^{-in\omega t}))$$

$$f(t) = \sum_{n=-N}^N c_n e^{in\omega t}$$

From periodic signals to continuous functions



$$f(t) = a_0 + \sum_{n=1}^N (a_n \cos(n\omega t) - b_n \sin(n\omega t)).$$



$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} a(\omega) e^{i\omega t} d\omega$$

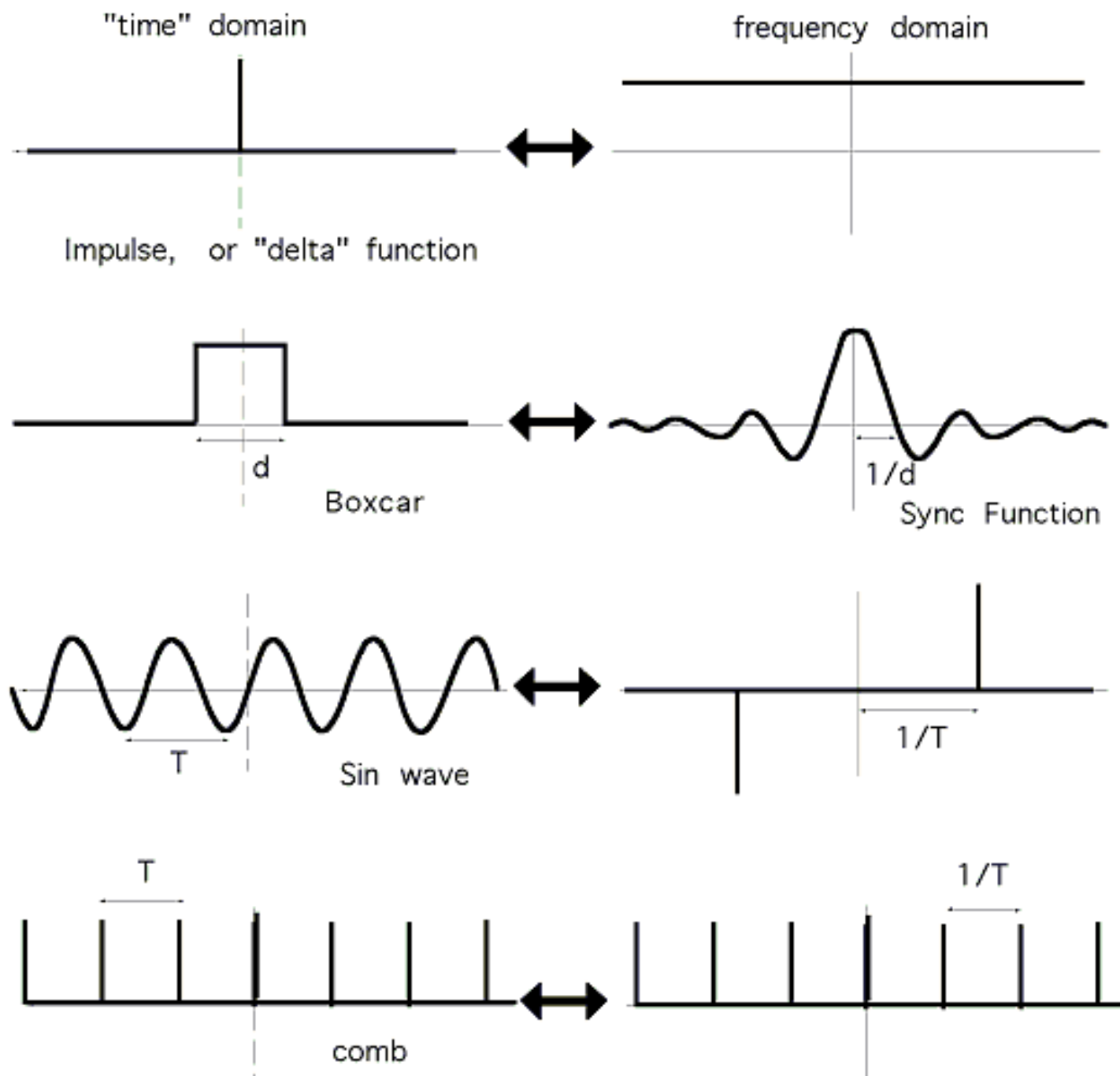
Spectrum has a real and an imaginary part !

From periodic signals to continuous functions

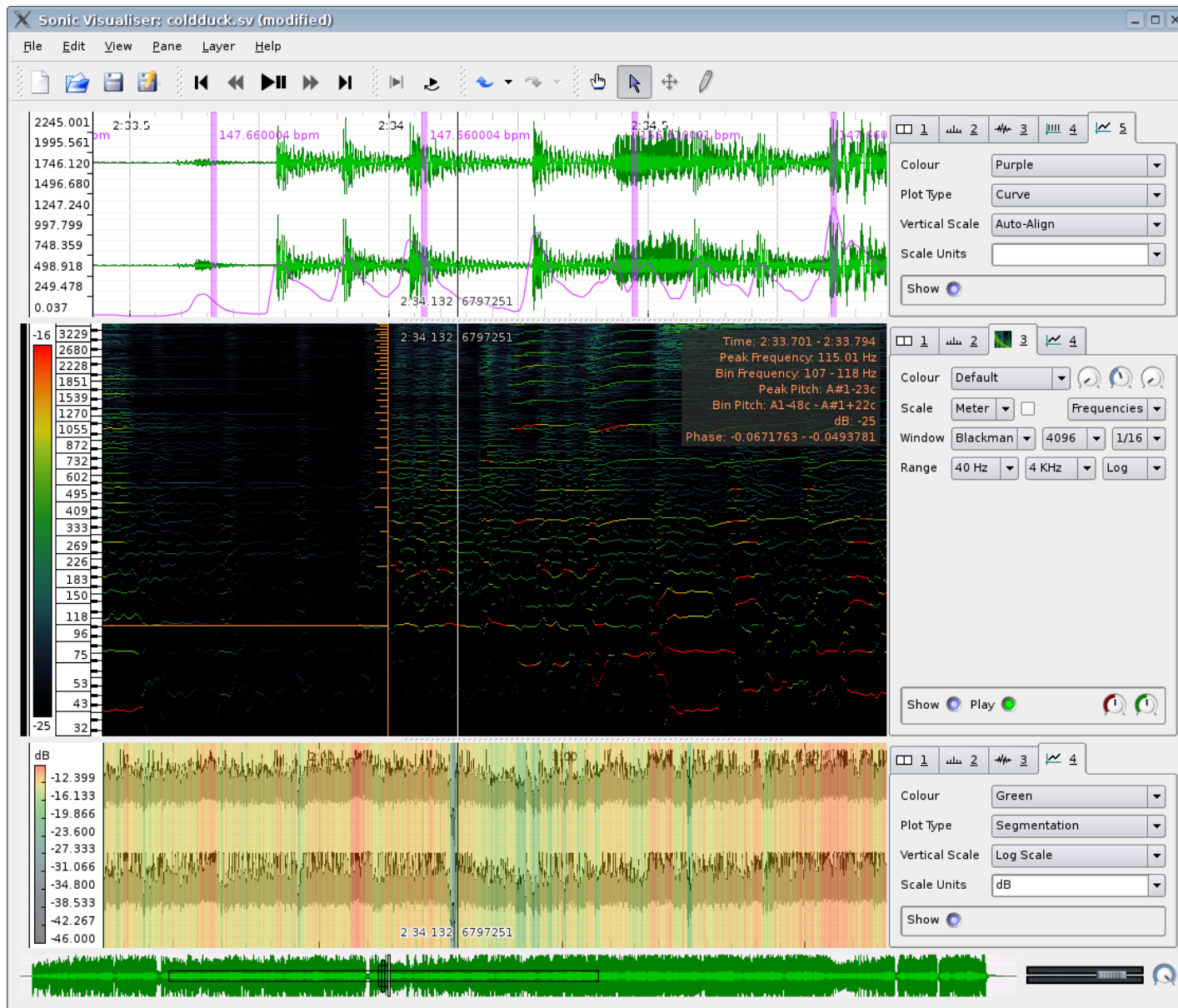
$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} a(\omega) e^{i\omega t} d\omega$$

$$a(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt.$$

Spectral Density: $\Phi(\omega) = a(\omega) \cdot \overline{a(\omega)}$

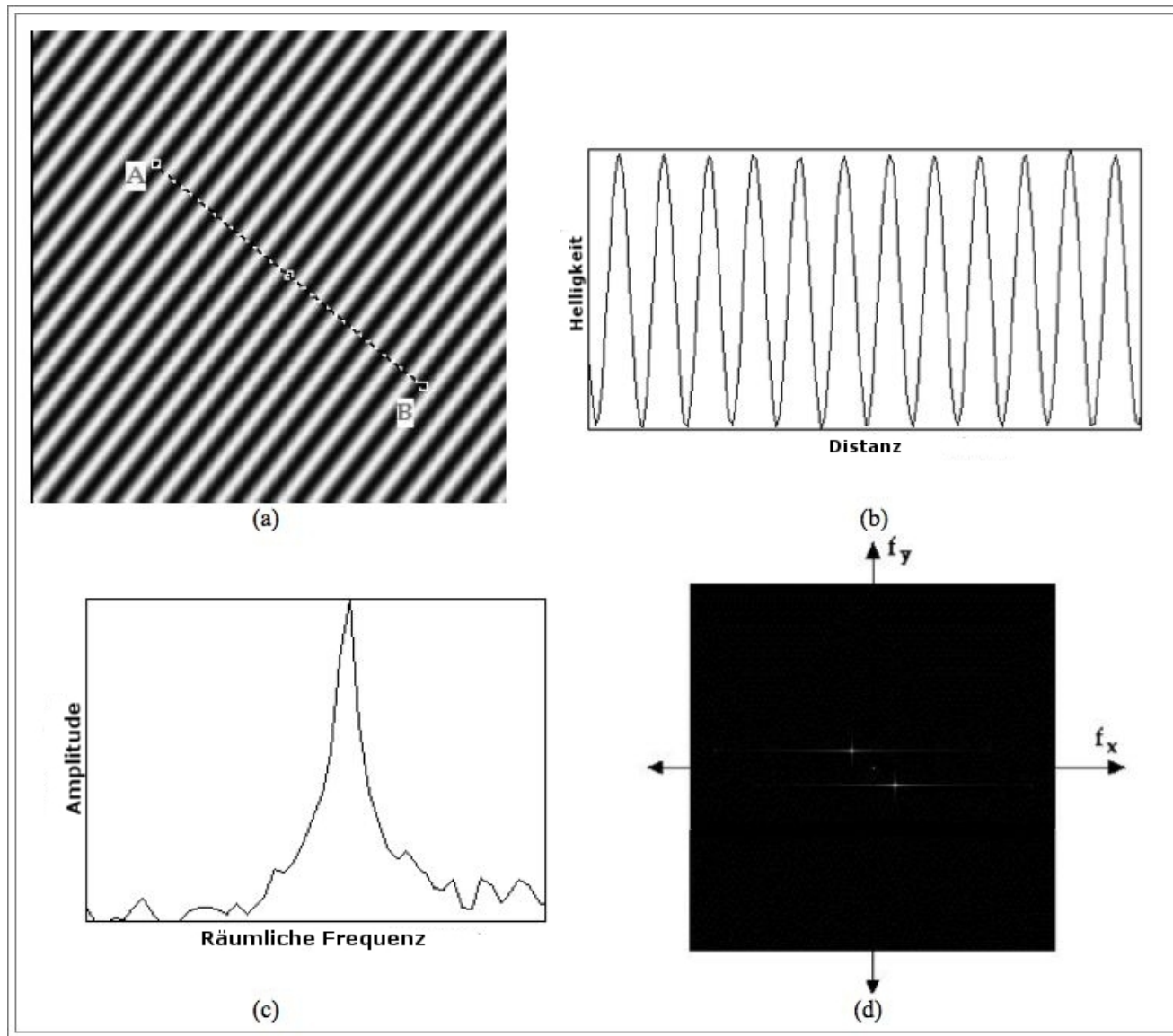


Typical Applications: Signal Analysis

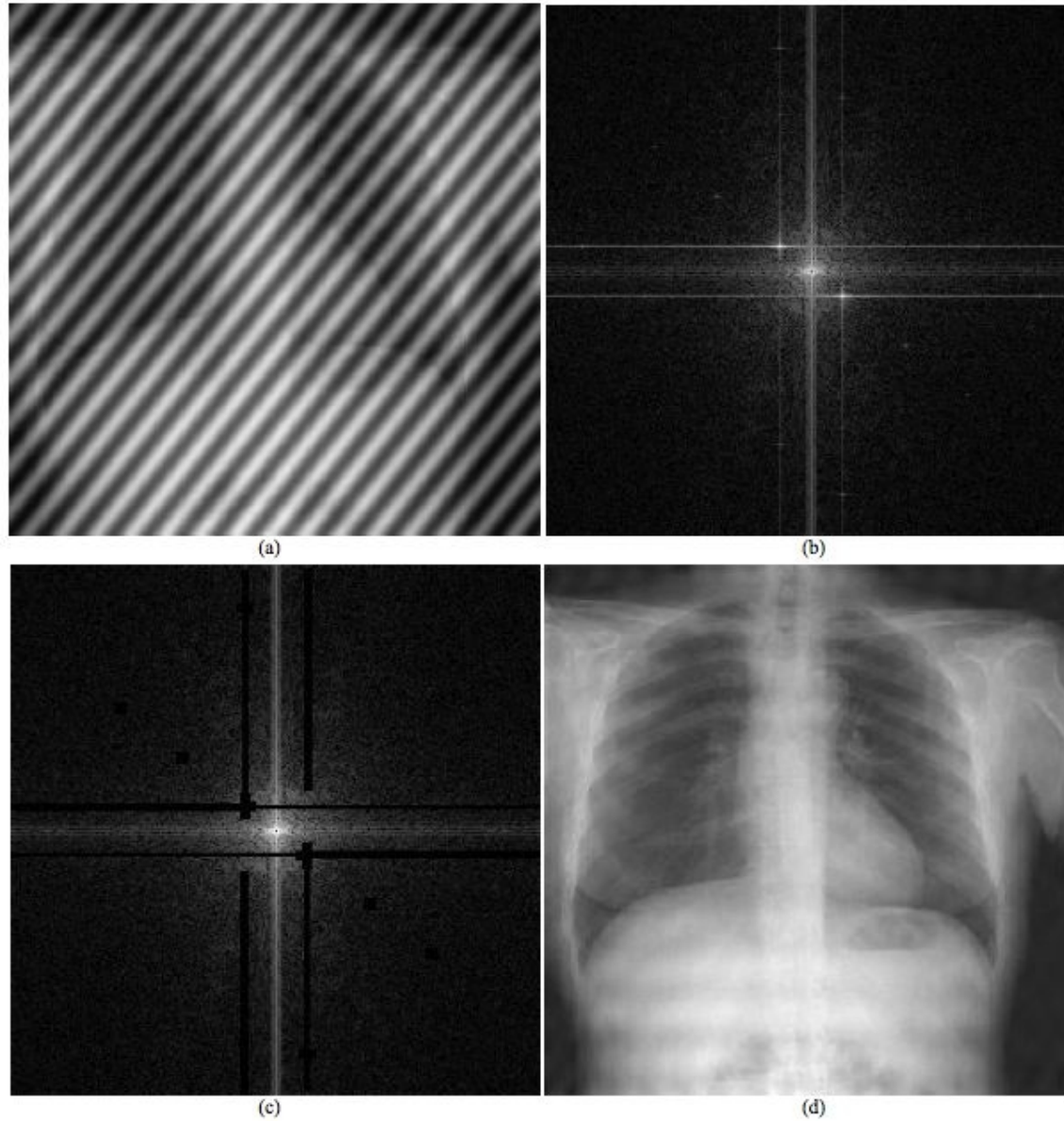


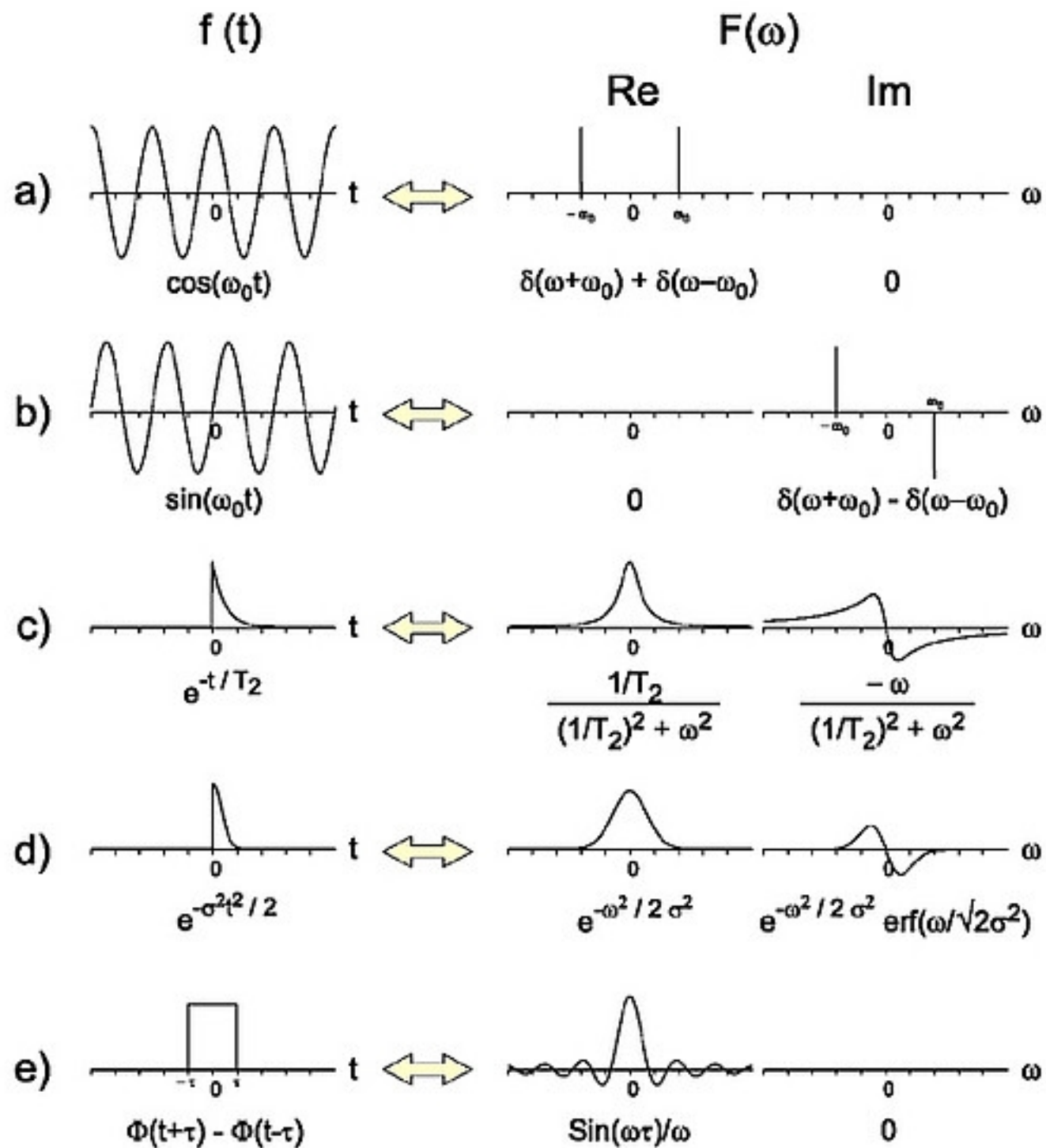
www.sonicvisualiser.org

Typical Applications: Image Processing

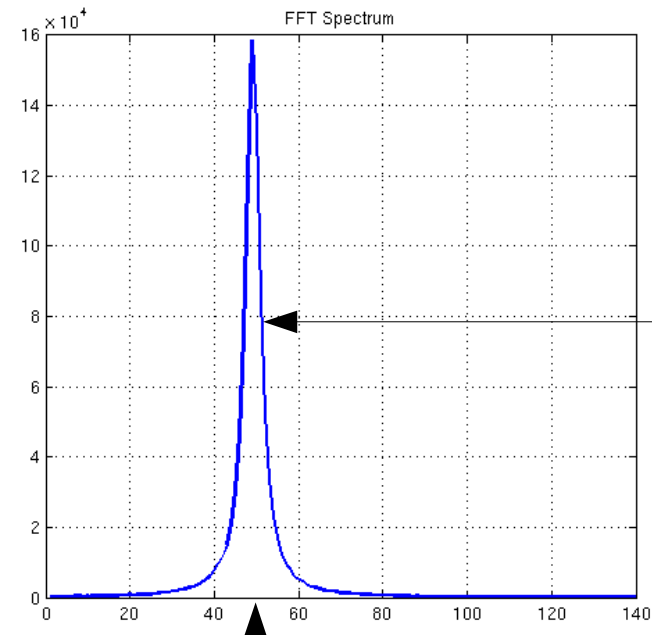
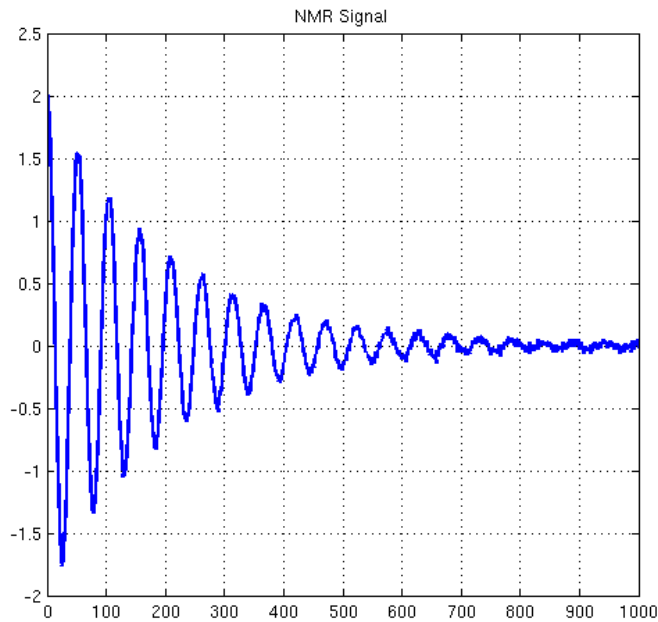


Typical Applications: Image Processing





Typical Applications: Signal Analysis

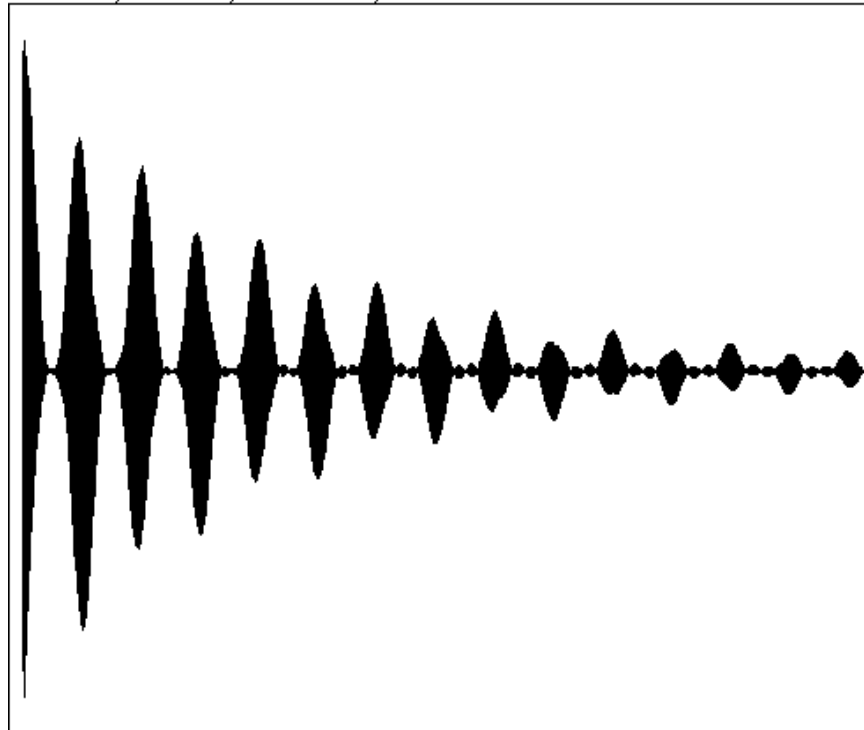


$$f(t) = e^{i\phi} e^{i\omega t} e^{\frac{t}{T_2}}$$

phase oscillation decay

The Sound of Chemistry

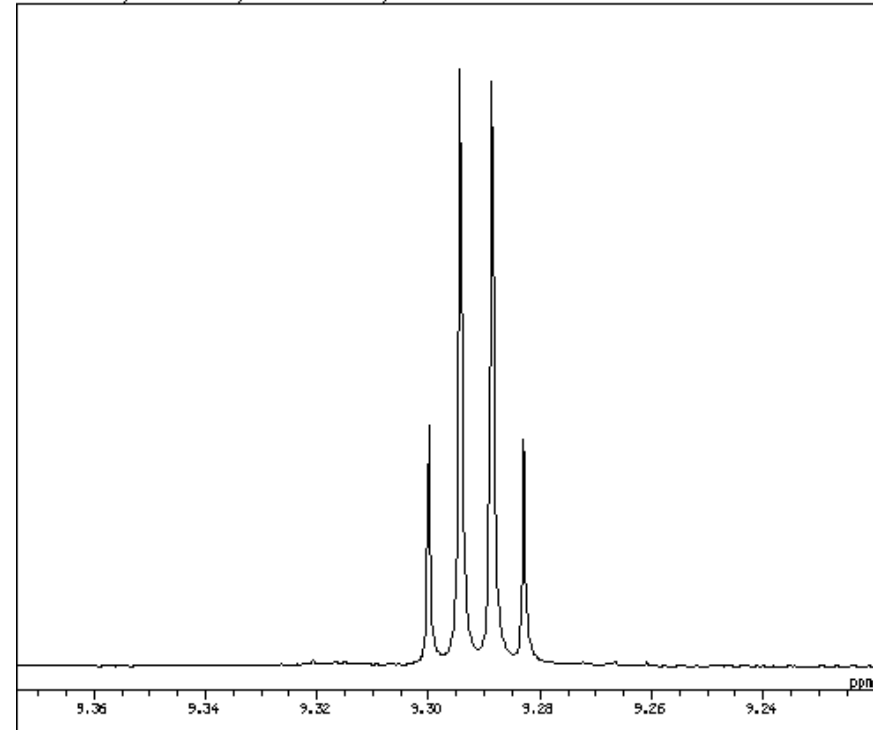
CH3CHO IN C6D6, KILMO SAMPLE, QUARTET SELECTIVE, 6000 GHZ



JEOL

L2-AUG-1996 18:22:13.86
 SFIL : CH3CHO.QUARTET
 EXMOD : SINGL
 IRMOD : NON
 POINT : 4096
 SAMPD : 4096
 FREQ : 800.00 Hz
 FILTR : 400 Hz
 SCANS : 4
 QUNTY : 0
 ACQTH : 5.1200 sec
 PD : 2.7232 sec
 RGAIN : 15
 PFI : 3.00 usec
 DEMLC : 1H
 DBFQ : 500.00 MHz
 DBSET : 164798.86 Hz
 IRMLC : 13C
 IRFQ : 125.65 MHz
 IRSET : 127988.00 Hz
 IRATN : 511
 IRAPM : 50.0 usec
 IRBP1 : 30
 IRBP2 : 6
 IRMS : 0
 TRMLC : 1H
 TRFQ : 500.00 MHz
 TRSET : 162410.00 Hz
 TRATN : 511
 TRAPM : 50.0 usec
 TRBP1 : 30
 TRBP2 : 6
 TRMS : 0
 CTEP : 23.5 c
 CSPE : 11 Hz
 SLINT : C6D6
 RESOL : 0.20 Hz
 WNOYD : 8
 BF : 0.10 Hz
 BF : 0.00 Hz
 PP : 3432 op
 ABSPO : 0.00 deg
 ABSF1 : 0.00 deg
 T1 : 0.00 %
 T2 : 0.00 %
 T3 : 90.00 %
 T4 : 100.00 %
 REFVL : 5.30 ppm
 THSP : 10000
 XE : 800.00 Hz
 XS : -14.06 Hz
 Y6 : 1.1421

CH3CHO IN C6D6, KILMO SAMPLE, QUARTET SELECTIVE, 6000 GHZ



JEOL

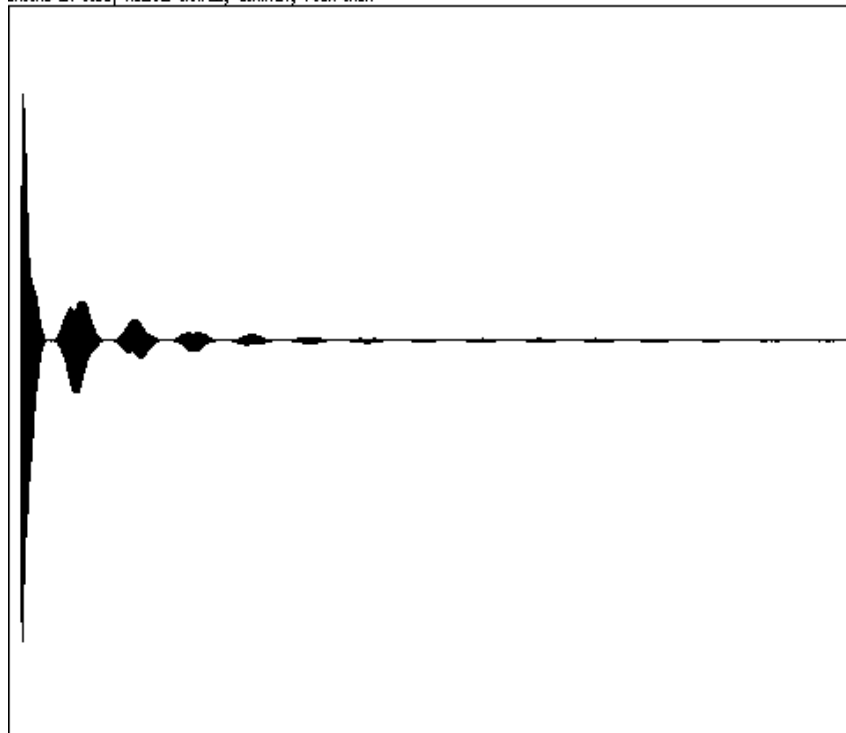
L2-AUG-1996 18:41:54.17
 SFIL : CH3CHO.QUARTET
 EXMOD : SINGL
 IRMOD : NON
 POINT : 8192
 SAMPD : 4096
 FREQ : 800.00 Hz
 FILTR : 400 Hz
 SCANS : 4
 QUNTY : 0
 ACQTH : 5.1200 sec
 PD : 2.7232 sec
 RGAIN : 15
 PFI : 3.00 usec
 DEMLC : 1H
 DBFQ : 500.00 MHz
 DBSET : 164798.86 Hz
 IRMLC : 13C
 IRFQ : 125.65 MHz
 IRSET : 127988.00 Hz
 IRATN : 511
 IRAPM : 50.0 usec
 IRBP1 : 30
 IRBP2 : 6
 IRMS : 0
 TRMLC : 1H
 TRFQ : 500.00 MHz
 TRSET : 162410.00 Hz
 TRATN : 511
 TRAPM : 50.0 usec
 TRBP1 : 30
 TRBP2 : 6
 TRMS : 0
 CTEP : 23.5 c
 CSPE : 11 Hz
 SLINT : C6D6
 RESOL : 0.10 Hz
 WNOYD : 8
 BF : 0.10 Hz
 BF : 0.00 Hz
 PP : 6821 op
 ABSPO : -90.00 deg
 ABSF1 : 0.00 deg
 T1 : 0.00 %
 T2 : 0.00 %
 T3 : 90.00 %
 T4 : 100.00 %
 REFVL : 5.30 ppm
 THSP : 6794
 XE : 77.13 Hz
 XS : 264.94 Hz
 Y6 : 0.0066

CH3CHO FID

CH3CHO Spectrum

The Sound of Chemistry

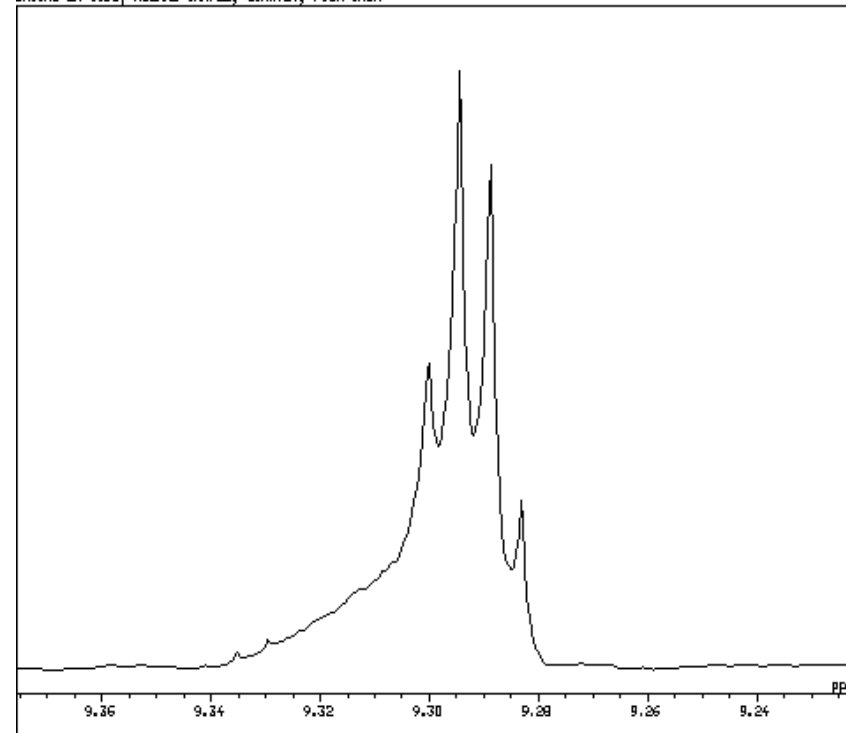
CH3CHO IN C6D6, HILAND SAMPLE, QUARTET, POOR SHIM



JEOL
 12-RUB-1996 18:11:42.67
 SFILE : CH3CHO_QUARTET_POOR
 EXMOD : SINGL
 IRMOD : NON
 PDIRT : 4096
 SAMPD : 4096
 FREQ : 900.00 Hz
 FILTR : 400 Hz
 SCANS : 4
 QUNIT : 0
 ACQTH : 5.1200 sec
 PD : 2.7222 sec
 READN : 18
 PH1 : 3.00 usec
 OBNJC : LH
 OBFREQ : 300.00 MHz
 OBSET : 164798.86 Hz
 IRNJC : 13C
 IREFRQ : 125.65 MHz
 IRESE : 127958.00 Hz
 IRTATN : 511
 IREFP1 : 50.0 usec
 IREFP2 : 30
 IREFP3 : 6
 IREFP4 : 0
 TRNJC : LH
 TRFREQ : 300.00 MHz
 TRSET : 162410.00 Hz
 TRTATN : 511
 TRFPW : 50.0 usec
 TRFP1 : 30
 TRFP2 : 6
 TRFMS : 0
 CTETP : 23.7 °
 CSFED : 12 Hz
 SLVNT : C6D6
 RESOL : 0.20 Hz
 WINDO : 8
 BF : 0.10 Hz
 BF : 0.00 Hz
 PF : 1 op
 ABSPO : 0.00 deg
 ABSP1 : 0.00 deg
 T1 : 0.00 s
 T2 : 0.00 s
 T3 : 90.00 s
 T4 : 100.00 s
 REFL : 9.30 ppm
 TREF : 6794
 XE : 900.00 Hz
 X8 : -13.09 Hz
 Y6 : 0.9976

CH3CHO FID

CH3CHO IN C6D6, HILAND SAMPLE, QUARTET, POOR SHIM



JEOL
 12-RUB-1996 18:15:48.80
 SFILE : CH3CHO_QUARTET_POOR
 EXMOD : SINGL
 IRMOD : NON
 PDIRT : 4096
 SAMPD : 4096
 FREQ : 900.00 Hz
 FILTR : 400 Hz
 SCANS : 4
 QUNIT : 0
 ACQTH : 5.1200 sec
 PD : 2.7222 sec
 READN : 18
 PH1 : 3.00 usec
 OBNJC : LH
 OBFREQ : 300.00 MHz
 OBSET : 164798.86 Hz
 IRNJC : 13C
 IREFRQ : 125.65 MHz
 IRESE : 127958.00 Hz
 IRTATN : 511
 IREFP1 : 50.0 usec
 IREFP2 : 30
 IREFP3 : 6
 IREFP4 : 0
 TRNJC : LH
 TRFREQ : 300.00 MHz
 TRSET : 162410.00 Hz
 TRTATN : 511
 TRFPW : 50.0 usec
 TRFP1 : 30
 TRFP2 : 6
 TRFMS : 0
 CTETP : 23.7 °
 CSFED : 12 Hz
 SLVNT : C6D6
 RESOL : 0.20 Hz
 WINDO : 8
 BF : 0.10 Hz
 BF : 0.00 Hz
 PF : 3432 op
 ABSPO : -42.13 deg
 ABSP1 : 0.00 deg
 T1 : 0.00 s
 T2 : 0.00 s
 T3 : 90.00 s
 T4 : 100.00 s
 REFL : 9.30 ppm
 TREF : 3402
 XE : 77.15 Hz
 X8 : 264.94 Hz
 Y6 : 0.0286

CH3CHO Spectrum

That's it!

References

- <http://de.wikipedia.org/wiki/Fourier-Transformation>
- http://en.wikipedia.org/wiki/Fourier_transform
- <http://www.chemie.uni-erlangen.de/oc/research/NMR/music.html>
- <http://www.sonicvisualiser.org>